On Private Peering Agreements between Content and Access Providers: A Contractual Equilibrium Analysis

XIN WANG and RICHARD T. B. MA, National University of Singapore, Singapore

Driven by the rapid growth of content traffic and the demand for service quality, Internet content providers (CPs) have started to bypass transit providers and connect with access providers directly via private peering agreements. This peering relationship often raises disputes, e.g., Netflix vs. Comcast, and is not well understood. In this paper, we build a peering contract model and propose the concept of *contractual equilibrium*, based on which we study the formation and evolution of peering contracts. By using market data, we emulate the strategic peering behavior of providers and shed light on the understanding of private peering agreements.

We reveal that the *superiority* and *market dominance* of providers primarily determine their peering strategies. We show that 1) superior providers tend to engage in peering more aggressively, and 2) non-dominant CPs' optimal peering strategies are negatively correlated due to market cannibalism, while the dominant CP often behaves oppositely. Our findings help explain phenomena such as why Netflix and Comcast signed the first peering contract, and reason whether private peering contracts will strengthen in future.

CCS Concepts: • **Networks** → **Network economics**; *Network performance modeling*; *Public Internet*.

Additional Key Words and Phrases: Internet; private peering; optimal contract; superiority; market dominance

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1 INTRODUCTION

Today's Internet is dominated by content traffic, especially video streaming. According to Sandvine 2018 Internet Phenomena Report [65], video is almost 58% of the total downstream Internet traffic, while Netflix solely contributes 15% of the downstream traffic across the entire Internet. To accommodate the increasing traffic demand from users, large content providers (CPs) have been deploying wide-area infrastructures so as to bring content closer to users and bypass transit providers on many paths [29]. For example, Netflix uses third-party content delivery networks, e.g., Akamai and Limelight, and builds its own [57]. This causes the *flattening phenomenon* [20, 29] of the Internet topology, which transitioned from a transit hierarchy to a peering mesh.

However, regardless how flat the Internet could be, end-users still rely on the last-mile access providers (APs) for accessing the Internet. Thus, the limited capacity of APs may constrain users' download speed due to network congestion. For example, the average throughput of Netflix users behind Comcast, the largest U.S. broadband provider, degraded 25% from over 2 Mbps in Oct 2013 to 1.5 Mbps in Jan 2014 [58]. Only after Netflix purchased a direct connection from Comcast to its network via a private peering agreement in Feb 2014 [76], did the average user throughput

Authors' address: Xin Wang, xin.wang@comp.nus.edu.sg; Richard T. B. Ma, tbma@comp.nus.edu.sg, School of Computing, National University of Singapore, Singapore, 117417.

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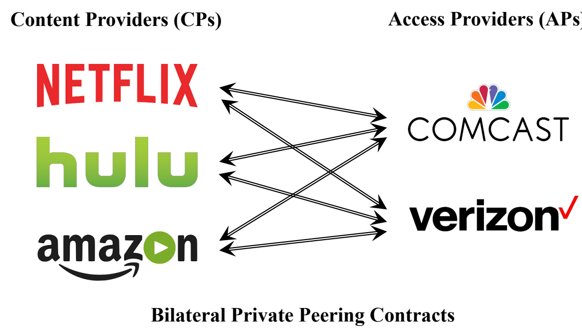

Fig. 1. Two-sided market with private peering.

rebounded almost doubly. Due to similar reasons, Netflix also signed another peering contract with Verizon two months later [31].

As a content service market, the entire Internet can be regarded as a two-sided platform illustrated in Figure 1, where CPs and APs may establish private peering agreements that affect the performance of content applications. Although from a service perspective, this peering relationship seems to be a collaborative effort between CPs and APs to improve service quality, the actual implementation is driven by the autonomous objectives and decisions of the contracting entities. In contrast to the traditional customer-provider relationship under which APs pay upstream entities transit fees, this type of peering often involves an AP imposing termination fees on a counterparty that specializes in content distribution and therefore, often causes peering disputes, e.g., Netflix-Comcast and Netflix-Verizon [48], in the bargaining process of contract negotiation. In the Netflix-Comcast dispute, for example, Netflix claimed that congestion was made intentionally by Comcast to force it to pay for direct connectivity; while Comcast claimed that Netflix was sending more traffic to transit providers than what they could accommodate and caused congestion. These peering disputes also raise policy concerns such as net neutrality [75], and therefore, the U.S. Federal Communication Commission (FCC) has requested the peering agreements Netflix signed with Comcast and Verizon [48] for investigations.

Nevertheless, private peering is not well understood and has not been explored much in the research community, partially due to the lack of public data sources of peering agreements, which are often confidential business contracts. The difficulty also lies in the fact that any CP's peering decision is driven by the competition in the content market, as well as the bilateral bargaining with an AP, during which financial transfers are involved and potential disputes could happen. Ultimately, any bilateral contract depends on not only the two trading parties but also all other possible contracts engaged by other pairs of providers in the entire service market.

In this paper, we build an analytical peering contract model (Sections 2 and 3) and propose the concept of *contractual equilibrium* (Definition 2.3), based on which we study the formation and evolution (Sections 4 and 5) of peering contracts. In particular, by using market data as inputs to the contract model, we emulate the strategic peering behavior of providers and try to answer the following questions: What kind of CP-AP pairs has strong incentives to sign the first peering contract? (Section 4.1) What is the best-response strategy of a peering pair if others sign peering contracts? (Section 4.2) How does a contractual equilibrium look like? (Section 4.3) How should a CP adapt its contracts with varying characteristics of itself and others? (Section 5.1) How does the contractual equilibrium evolve when conditions of market environment change? (Section 5.2)

We reveal that the *superiority* and *market dominance* (Definitions 3.2 and 3.3) of providers play key roles in determining their optimal peering strategies. The former is defined by two intrinsic properties of providers, while the latter is determined by whether a CP obtains more than half of the entire user market. Some of our major findings are as follows.

(1) A CP-AP pair is more likely to sign the first peering contract if they are superior to their competitors. In general, superior providers also tend to maintain contracts at high levels and obtain high market shares.
(2) As the market penetration and quality requirements increase for the Internet content, non-dominant CPs will strengthen their peering contracts, while a CP will behave oppositely after it dominates the market.
(3) In response to enhanced peering relationship between other providers, non-dominant CPs will weaken their peering contracts due to market cannibalism. However, a dominant CP will behave oppositely.
(4) As a CP becomes superior and dominant, most peering contracts, even its own, will weaken.

Our findings shed light upon the understanding of private peering agreements between content and access providers.

2 CONTRACT MODEL & EQUILIBRIUM

We consider an Internet content market such as online video streaming. It consists of a set $\mathcal{M}$ of content providers (CPs) which compete with each other for end-users. To provide content to users, CPs need to deliver content data via the access providers (APs) that users subscribe to. We denote the set of the APs by $\mathcal{N}$. As APs deliver data traffic from CPs to users, CPs and APs may wish to establish private peering through which users can obtain better service quality. In our model, private peering refers to that APs improve the data delivery quality of CPs for a fee. Our model does not restrict the type of resources used by the AP to improve the quality, for example, the AP could deploy memory storage to cache data for the CP or dedicate bandwidth resources to build a direct connection with the CP like Netflix-Comcast [76]. We denote the set of CP-AP pairs that could potentially establish private peering and jointly serve end-users by $\mathcal{L} = \mathcal{M} \times \mathcal{N}$.

As a key to understanding the private peering between CPs and APs lies in the contracts they sign, we model the contract terms of a CP-AP pair $l \in \mathcal{L}$ by a tuple (ϕ_l, ψ_l) of contractual level and monetary transfer, respectively. Because CP-AP peering often involves a CP paying an AP for the improvement of data delivery quality, where the AP needs to dedicate resources such as bandwidth and cache, the contractual level ϕ_l models the amount of resource the AP deploys under the contract and reflects the data delivery quality provided by the AP. The monetary transfer ψ_l denotes the corresponding contractual payment from the CP to the AP. We normalize $\phi_l \in [0, 1]$ without loss of generality. $\phi_l = 1$ models a contract under which abundant resources are deployed to establish a best-quality private peering, for example, abundant bandwidths are deployed to build a congestion-free interconnection between the AP and the CP or abundant caches are used to ensure a 100% of cache hit ratio of the CP's content. $\phi_l = 0$ models the case that no peering contract is signed. We define $\boldsymbol{\phi} = (\phi_l : l \in \mathcal{L})$ and $\boldsymbol{\psi} = (\psi_l : l \in \mathcal{L})$ to be the vectors of all contractual levels and transfers. We denote the entire *contract profile* by $(\boldsymbol{\phi}, \boldsymbol{\psi})$ and the profile except pair l's contract by $(\boldsymbol{\phi}_{-l}, \boldsymbol{\psi}_{-l})$.

Because any contractual level ϕ_l affects the service quality for users who choose the pair l of providers, it influences the choices of users among alternative providers and therefore affects the market share and profits of providers. Nonetheless, as profit-seeking business entities, CPs and APs negotiate the terms of peering contract via a bargaining process driven by their individual profits. We denote the profits of any CP $m \in \mathcal{M}$ and AP $n \in \mathcal{N}$ by $P_m(\boldsymbol{\phi}, \boldsymbol{\psi})$ and $Q_n(\boldsymbol{\phi}, \boldsymbol{\psi})$, which are functions of the entire contract profile $(\boldsymbol{\phi}, \boldsymbol{\psi})$, because a provider's profit may depend on not only the contracts it signs but also the contracts other providers have reached. In particular, we define the profits of any CP m and AP n by

$$P_m(\boldsymbol{\phi}, \boldsymbol{\psi}) \triangleq U_m(\boldsymbol{\phi}) - \sum_{k \in \mathcal{L}^m} \psi_k; \; Q_n(\boldsymbol{\phi}, \boldsymbol{\psi}) \triangleq \sum_{k \in \mathcal{L}_n} \psi_k - V_n(\boldsymbol{\phi}), \tag{1}$$

where $\mathcal{L}^m \triangleq \{m\} \times \mathcal{N}$ and $\mathcal{L}_n \triangleq \mathcal{M} \times \{n\}$ define the set of CP-AP pairs involving CP m and AP n, respectively, and $U_m(\boldsymbol{\phi})$ and $V_n(\boldsymbol{\phi})$ denote the CP's revenue and AP's cost, respectively. Here, the profit of a CP is defined as its revenue (from users and/or advertisers) minus the payments to APs, while the profit of an AP is defined as its received payments from CPs minus its cost for fulfilling the peering contracts[1]. Because both the CP's revenue and AP's peering cost are not affected by the payments among CPs and APs, $U_m(\boldsymbol{\phi})$ and $V_n(\boldsymbol{\phi})$ are merely functions of the contractual levels $\boldsymbol{\phi}$.

To understand what peering contracts would be signed between CPs and APs, we first define an *optimal contract* that is ideally desirable for a pair of CP and AP to reach.

Definition 2.1 (Optimal Contract). For any $l = (m, n) \in \mathcal{L}$, given the contract profile $(\boldsymbol{\phi}_{-l}, \boldsymbol{\psi}_{-l})$ of all other CP-AP pairs, the contract (ϕ_l, ψ_l) is optimal if two conditions are satisfied:
1) [Profit Maximization] the contractual level ϕ_l maximizes the aggregate profit of CP m and AP n, i.e., for any $\bar{\phi}_l \in [0, 1]$,

$$P_m(\boldsymbol{\phi}, \boldsymbol{\psi}) + Q_n(\boldsymbol{\phi}, \boldsymbol{\psi}) \geq P_m(\bar{\phi}_l, \boldsymbol{\phi}_{-l}, \boldsymbol{\psi}) + Q_n(\bar{\phi}_l, \boldsymbol{\phi}_{-l}, \boldsymbol{\psi});$$

2) [Fair Profit Sharing] the contractual transfer ψ_l grants equal profit gains to CP m and AP n, i.e.,

$$P_m(\boldsymbol{\phi}, \boldsymbol{\psi}) - P_m(0, \boldsymbol{\phi}_{-l}, 0, \boldsymbol{\psi}_{-l}) = Q_n(\boldsymbol{\phi}, \boldsymbol{\psi}) - Q_n(0, \boldsymbol{\phi}_{-l}, 0, \boldsymbol{\psi}_{-l}).$$

Definition 2.1 states that an optimal contract between a CP-AP pair is defined based on two axiomatic conditions imposed on the level ϕ_l and transfer ψ_l of contract, respectively. The first condition requires that the contractual level ϕ_l maximizes the aggregate profit $P_m + Q_n$, which does not depend on the internal transfer ψ_l. This condition is desirable for an optimal contract since it guarantees that the contract outcome is *Pareto optimal* for the CP-AP pair, i.e., there does not exist another contractual level under which one provider's profit can be increased without reducing that of the other. Because bargaining will lead to a Pareto efficient outcome as stated by Coase Theorem [13], the most probably signed contract should induce such an efficient outcome. The second condition requires that the contractual payment ψ_l equalizes the profit gain, i.e., the difference in profit derived from signing a contract at level ϕ_l, for both providers. This condition is desirable since it guarantees the fairness, in the sense of both Egalitarian bargaining solution [42] and the Shapley value [67], between the CP and AP. Furthermore, such a fair payment has also been shown to be the solution of strategic bilateral bargaining of alternating offers [7, 64], which demonstrates the practicality and stability of the solution. In summary, the optimal contract of a CP-AP pair is a stable bargaining solution between the CP and the AP. In other words, a CP-AP pair does not have incentives to change its contract term, e.g., contractual level or monetary transfer, under the optimal contract.

As the domain of contractual level, i.e., $[0, 1]$, is compact, the existence of an optimal contract is trivial. Next, we provide the condition for the uniqueness of an optimal contract and characterize the optimal contractual level and transfer in terms of the revenue $U_m(\boldsymbol{\phi})$ of CP and cost $V_n(\boldsymbol{\phi})$ of AP.

[1] Although an AP's actual profit also include its revenue from residential users of Internet access, for the purpose of analyzing peering contracts, we focus on the components of profit that influence an AP's peering decisions.

Theorem 2.2 (Characterization of Optimal Contract). *For any CP-AP pair $l = (m, n) \in \mathcal{L}$, if $W_l(\boldsymbol{\phi}) \triangleq U_m(\boldsymbol{\phi}) - V_n(\boldsymbol{\phi})$ is strictly concave in the contractual level ϕ_l, there always exists a unique optimal contract (ϕ_l^*, ψ_l^*) that satisfies*

$$\phi_l^* = \arg\max_{\bar{\phi}_l \in [0,1]} \left\{ U_m(\bar{\phi}_l; \boldsymbol{\phi}_{-l}) - V_n(\bar{\phi}_l; \boldsymbol{\phi}_{-l}) \right\} \quad \textit{and}$$
$$\psi_l^* = \frac{1}{2} \left[U_m(\phi_l^*, \boldsymbol{\phi}_{-l}) - U_m(0, \boldsymbol{\phi}_{-l}) + V_n(\phi_l^*, \boldsymbol{\phi}_{-l}) - V_n(0, \boldsymbol{\phi}_{-l}) \right].$$

Theorem 2.2 shows that the uniqueness of optimal contract can be guaranteed by the concavity of the utility W_l, defined as the revenue of CP m minus the cost of AP n. This concavity can be justified by the concavity of CP's revenue U_m due to diminishing returns and the convexity of AP's cost V_n due to increasing marginal costs. It further characterizes that 1) the optimal level ϕ_l^* that maximizes providers' total profits $P_m + Q_n$ needs to be the one that maximizes the utility $W_l = U_m - V_n$, and 2) the optimal transfer ψ_l^* that equalizes changes in profits needs to be half of the sum of changes in CP's revenue U_m and AP's cost V_n after the peering contract is fulfilled at level ϕ_l^*. The latter characterization could be understood intuitively since the CP needs to share half of its additional revenue and half of the incurred cost at the AP, which is aligned with previous work [16, 32, 77] that use Nash Bargaining solution to specify fair payment transfers.

Based on our characterization of an optimal contract, we define the concept of *contractual equilibrium* that specifies what contract profile will be formed between all CP-AP pairs.

Definition 2.3 (Contractual Equilibrium). A contract profile $(\boldsymbol{\phi}, \boldsymbol{\psi})$ is a *contractual equilibrium* if for any CP-AP pair $l \in \mathcal{L}$, (ϕ_l, ψ_l) is an optimal contract given the contracts $(\boldsymbol{\phi}_{-l}, \boldsymbol{\psi}_{-l})$ of others.

Definition 2.3 embodies the essence of a Nash equilibrium. Instead of characterizing the strategy profile of players in the context of a pure non-cooperative game, contractual equilibrium characterizes the profile $(\boldsymbol{\phi}, \boldsymbol{\psi})$ of bilateral contracts. Similar to the Nash equilibrium, a contractual equilibrium is defined for any bilateral contract (ϕ_l, ψ_l), which specifies the conditions under which the parties $l = (m, n)$ reach to their optimal contract and thus will not have incentives to "unilaterally" change the contract. This ensures the stability of contracts specified under equilibrium.

Notice that Definition 2.3 is defined for any single contract and does not make any assumption or restriction on whether multiple contracts of different CP-AP pairs can be changed simultaneously. In reality, as each CP's strategy includes the set of contracts with all the APs, each CP might want to simultaneously change multiple contracts. If characterizing a CP's strategy that is resistant to group deviation rather than unilateral deviation in terms of peering contract, the first condition in Definition 2.1 needs to be strengthened to allow simultaneous changes of multiple contractual levels for any CP. We call such a strategy profile an *equilibrium under group deviation.* Because an equilibrium under group deviation must satisfy a stronger version of the first condition in Definition 2.1, it must satisfy all the conditions for a contractual equilibrium. Although a contractual equilibrium might be of a less stable solution, it is a necessary condition of being an equilibrium and is more tractable. In particular, we will characterize the existence and uniqueness of contractual equilibrium and study its comparative statics. Nevertheless, since an equilibrium under group deviation is also a contractual equilibrium, its comparative statics and the corresponding insights should be aligned with what holds for contractual equilibrium.

Besides, due to the axiomatic conditions imposed on an optimal contract for bilateral bargaining in Definition 2.1, our definition of contractual equilibrium also constitutes a "Nash equilibrium in Nash bargains" solution, i.e., separate bilateral Nash bargaining problems within a Nash equilibrium to a game played among all pairs of firms, which is referred to as a "Nash-in-Nash" solution.

Corollary 2.4. *If a contract profile $(\boldsymbol{\phi}, \boldsymbol{\psi})$ is a contractual equilibrium, it is a Nash-in-Nash solution, i.e., for any pair $l = (m, n) \in \mathcal{L}$, given the contracts of other pairs $(\boldsymbol{\phi}_{-l}, \boldsymbol{\psi}_{-l})$, the contract*

(ϕ_l, ψ_l) is a Nash bargaining solution between the CP m and the AP n, i.e., (ϕ_l, ψ_l) maximizes the Nash product $[P_m(\boldsymbol{\phi}, \boldsymbol{\psi}) - P_m(0, \boldsymbol{\phi}_{-l}, 0, \boldsymbol{\psi}_{-l})]\,[Q_n(\boldsymbol{\phi}, \boldsymbol{\psi}) - Q_n(0, \boldsymbol{\phi}_{-l}, 0, \boldsymbol{\psi}_{-l})]$ subject to the constraints $P_m(\boldsymbol{\phi}, \boldsymbol{\psi}) \geq P_m(0, \boldsymbol{\phi}_{-l}, 0, \boldsymbol{\psi}_{-l})$ and $Q_n(\boldsymbol{\phi}, \boldsymbol{\psi}) \geq Q_n(0, \boldsymbol{\phi}_{-l}, 0, \boldsymbol{\psi}_{-l})$.

"Nash-in-Nash" solution has been widely studied and used in the literature[2] of industrial organization [8, 14, 22, 37], since it provides easily computable payments for complicated bilateral oligopoly environments with interdependencies, and it is based on marginal valuations which fits well with classical price theory [14].

The following result characterizes the existence and uniqueness of contractual equilibrium, again based on conditions of the utility $W_l(\boldsymbol{\phi}) \triangleq U_m(\boldsymbol{\phi}) - V_n(\boldsymbol{\phi})$ of CP-AP pairs.

Theorem 2.5 (Existence and Uniqueness). *There always exists at least one contractual equilibrium, if $W_l(\boldsymbol{\phi}) \triangleq U_m(\boldsymbol{\phi}) - V_n(\boldsymbol{\phi})$ is quasi-concave in ϕ_l for any pair $l \in \mathcal{L}$. Furthermore, the equilibrium is unique, if for any two distinct contractual levels $\boldsymbol{\phi}' \neq \boldsymbol{\phi}$, there exists a pair l such that*

$$(\phi_l' - \phi_l)\left[\frac{\partial W_l(\boldsymbol{\phi}')}{\partial \phi_l'} - \frac{\partial W_l(\boldsymbol{\phi})}{\partial \phi_l}\right] < 0. \tag{2}$$

Theorem 2.5 shows that the existence of contractual equilibrium is guaranteed under the condition of quasi-concavity on the utility W_l of pairs. The sufficient condition for the uniqueness of contractual equilibrium requires the negative of marginal utility, i.e., $-\partial W_l(\boldsymbol{\phi})/\partial \phi_l$, to be a P-function [52].

3 REVENUE, COST AND CHOICE MODEL

In the previous section, we modeled the peering contracts between CPs and APs and formulated the concepts of optimal contract and contractual equilibrium. Our analysis showed that both solution concepts are fundamentally determined by CP's revenue $U_m(\boldsymbol{\phi})$ and AP's cost $V_n(\boldsymbol{\phi})$. In this section, we construct the revenue and cost of providers based on 1) detailed characteristics of the CPs and APs, and 2) the user choices over the providers impacted by peering contracts.

We denote the total user population that consumes content by X. Based on the CP's business model, its revenue can be generated from subscription fees of users, e.g., Netflix, or advertisement fees generated from user views, e.g., YouTube. We denote the average per-user revenue of CP m by u_m. Similarly, as private peering requires APs to invest and deploy resources such as bandwidth and cache capacities, we denote the resource cost of AP n at the highest contract level by v_n, i.e., the cost for fulfilling a best-quality private peering. Notice that although the bandwidth cost/price of paid peering has declined by 20-30% annually in recent years similarly to those of transit prices [1], the volume of data traffic generated by CPs has increased by 20-30% annually [71] and many emerging content services, e.g., live streaming and HD online video, are highly sensitive to delay/throughput. Therefore, the bandwidth of peering needed by CPs to guarantee their service quality is expected to grow by at least 20-30% per year. Because the price of paid peering is often charged based on the bandwidth, we expect that total bandwidth cost for peering will not decline over the next few years. We define the revenue and cost of providers by

$$U_m(\boldsymbol{\phi}) \triangleq u_m \sum_{l \in \mathcal{L}^m} \pi_l(\boldsymbol{\phi}) X \quad \text{and} \quad V_n(\boldsymbol{\phi}) \triangleq v_n \sum_{l \in \mathcal{L}_n} \phi_l, \tag{3}$$

[2] Although the concept of the Nash-in-Nash solution appeared in literature, our work uses different models from them. For example, [14, 22] and [8, 37] modeled the contract term by a monetary transfer and a tuple of price and quantity of goods, respectively, while we model the contract term by a tuple of monetary transfer and contractual level of private peering.

where $\pi_l(\boldsymbol{\phi})$ defines the market share of CP m under the peering pair l, which depends on the contractual levels $\boldsymbol{\phi}$ of all pairs because users' choices among pairs are affected by the service quality influenced by the contract levels $\boldsymbol{\phi}$.

To characterize $\pi_l(\boldsymbol{\phi})$, we start with an ideal baseline case under which any CP m can be accessed from any AP n via a best-quality private peering. If users' choices over CPs are consistent with respect to the independence of irrelevant alternatives (IIA) property[3], by Luce's axiom [47], the probability of a user choosing CP m should be proportional to a weight associated with CP m. We denote this weight by α_m, and because the user's choice of AP does not affect the quality of CPs' services under this ideal scenario, α_m captures the intrinsic properties of CPs such as brand name and content coverage that influence users' choices over CPs. Besides, users who are interested in video content may also choose offline alternatives, e.g., DVDs. We conceptually model all these *outside options* as a competing content service provided by a CP index by $0 \in \mathcal{M}$. Likewise, we denote the weight of any AP n by β_n, which captures its intrinsic characteristics such as brand and price. To this end, we can understand that any user will choose CP m and AP n with probabilities $\alpha_m/(\sum_{i\in\mathcal{M}}\alpha_i)$ and $\beta_n/(\sum_{j\in\mathcal{N}}\beta_j)$, which are also the market share of the providers. Furthermore, if we regard a pair $l=(m,n)$ of CP and AP as a bundle of two complementary services, by extending Luce's axiom in a two-dimensional space, we know that each user will choose the pair of providers with a probability proportional to $\alpha_m\beta_n$. As a result, we can derive the market share π_l of any pair l under the ideal case of $\boldsymbol{\phi}=\mathbf{1}$, i.e., all contracts provide best-quality private peering, as $\pi_l(\mathbf{1})=\alpha_m\beta_n/(\sum_{l=(i,j)\in\mathcal{L}}\alpha_i\beta_j)$.

Next, we characterize the general market share $\pi_l(\boldsymbol{\phi})$ under any vector $\boldsymbol{\phi}$ of contractual levels. Intuitively, when a pair l does not maintain a best-quality private peering, i.e., $\phi_l < 1$, users' experience will be degraded and therefore, the chance for them to choose the pair of providers will decrease. We define the pair $l=(m,n)$'s weight under a contractual level $\phi_l\in[0,1]$ by $\alpha_m\beta_nG(\phi_l)$, i.e., the baseline weight $\alpha_m\beta_n$ multiplied by a gain factor $G(\phi_l)$ that satisfies $G(1)=1$ and decreases as ϕ_l decreases. The monotonicity of G guarantees that the probability of a pair being chosen by users increases with its contractual level and decreases with that of another pair, which models that a pair of CP-AP would attract (lose) more users when they (another pair of CP-AP) sign a contract of a higher level. Given the contractual levels $\boldsymbol{\phi}$ of all pairs[4], the expression of the probability that a user chooses a CP-AP pair $l=(m,n)$ can be generalized as

$$\pi_l(\boldsymbol{\phi}) = \frac{\alpha_m\beta_nG(\phi_l)}{\sum_{k=(i,j)\in\mathcal{L}}\alpha_i\beta_jG(\phi_k)}. \tag{4}$$

Corollary 3.1. *If the gain function $G(\cdot)$ is strictly concave, 1) there exists a unique contractual equilibrium and 2) for any $l\in\mathcal{L}$, given the contract profile $(\boldsymbol{\phi}_{-l},\boldsymbol{\psi}_{-l})$ of all other CP-AP pairs, there exists a unique optimal contract (ϕ_l^*,ψ_l^*).*

Corollary 3.1 shows that by a minor concavity assumption on the gain function $G(\cdot)$, the uniqueness of optimal contracts and contractual equilibrium can be guaranteed. Figure 2 illustrates a gain function in the form of $G(\phi_l)=1-g(1-\phi_l)^2$ that increases concavely with ϕ_l, which models the diminishing return of user preference. The parameter g captures the discount in link l's weight when its quality cannot be guaranteed without any peering contract signed, i.e., $\phi_l=0$. As shown by Theorem 2.2 that an optimal contract of a CP-AP pair $l\in\mathcal{L}$ is a function of $\boldsymbol{\phi}_{-l}$, we denote the optimal contract level and transfer by $\phi_l^*(\boldsymbol{\phi}_{-l})$ and $\psi_l^*(\boldsymbol{\phi}_{-l})$, respectively. We denote the unique contractual equilibrium by $(\boldsymbol{\phi}^*,\boldsymbol{\psi}^*)$.

[3]It requires the probability of choosing one over another from a set not to be affected by the presence or absence of other alternatives in the set.

[4]As the quality of the outside options does not rely on any AP, the contract level between CP 0 and any AP equals 1 by definition.

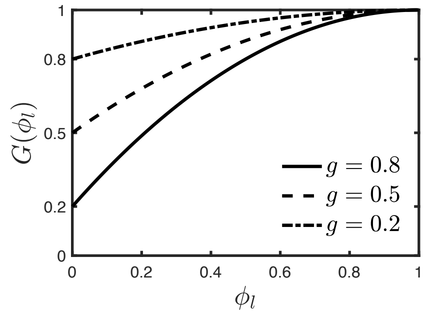

Fig. 2. Gain $G(\phi_l) = 1 - g(1 - \phi_l)^2$ under varying contract level ϕ_l with different values of parameter g.

So far, we have completed the descriptions of our model. In summary, each CP m and AP n are modeled by two intrinsic parameters (α_m, u_m) and (β_n, v_n) that are exogenous to the model. Based on a choice model of users, the peering contracts will determine endogenous variables such as the market share π_l of providers. In the following, we define two key concepts that are related to two intrinsic properties of the providers and the endogenous market share of them. We will show that both concepts play key roles in determining what contracts will be signed among CPs and APs.

Definition 3.2 (Binary Superiority Relation Of Providers). For any two CPs m and i, we define m to be at least as superior as i, denoted by $m \succcurlyeq i$, if and only if $\alpha_m \geq \alpha_i$ and $u_m \geq u_i$. For any two APs n and j, we define n to be at least as superior as j, denoted by $n \succcurlyeq j$, if and only if $\beta_n \geq \beta_j$ and $v_n \leq v_j$.

Definition 3.3 (Market Dominance). For any CP $m \in \mathcal{M}$, we denote its total market share by π_m, defined by $\pi_m(\boldsymbol{\phi}) \triangleq \sum_{l \in \mathcal{L}^m} \pi_l(\boldsymbol{\phi})$. We define a CP m to be *dominant* if its market share is no smaller than half of the market, i.e., $\pi_m \geq 50\%$.

Definition 3.2 defines a binary relation between providers that specifies a *partial ordering* of them. Intuitively, a superior provider has more incentives to sign contracts. However, this intuition does not always hold, as we will see that the conclusions may totally change depending on whether a CP is dominant as specified in Definition 3.3.

4 FORMATION OF PEERING CONTRACTS

Motivated by the emergence of CP-AP peering, in this section, we investigate the strategic behaviors of the providers and we start with a pragmatic approach that uses market data as inputs to our model and emulate the peering decisions of the providers. We consider the representative market structure in Figure 1 that consists of the largest three CPs, i.e., Nexflix, Hulu and Amazon Prime Video, and two major U.S. broadband APs, i.e., Comcast and Verizon. Because the first of such a peering contract [76] was signed in Feb 2014, we choose the model parameters based on the 2013 year-end market data as follows. CPs' weights are set to be $(\alpha_0, \alpha_1, \alpha_2, \alpha_3) = (24.3, 83.4, 39.5, 28.5)$ based on the fact that the Internet content market saturation was 75.7% in 2013 [68] and the CPs' market share ratio [63] was $38 : 18 : 13$. APs' weights are set to be $(\beta_1, \beta_2) = (20.7, 6.1)$ which were the numbers (in millions) of their users [12, 73] in 2013. The user population is set to be $X = \beta_1 + \beta_2 = 26.8$ millions. Based on the annual revenues and numbers of users of Netflix [56] and Hulu [38], their per-user revenues are set to be $(u_1, u_2) = (130.76, 63.17)$ dollars. Because Amazon Prime Video started to operate independently from 2016 [51], we use the ratio of its subscription fee and that of Netflix at 2016 and project its per-user revenue as 90% of that of

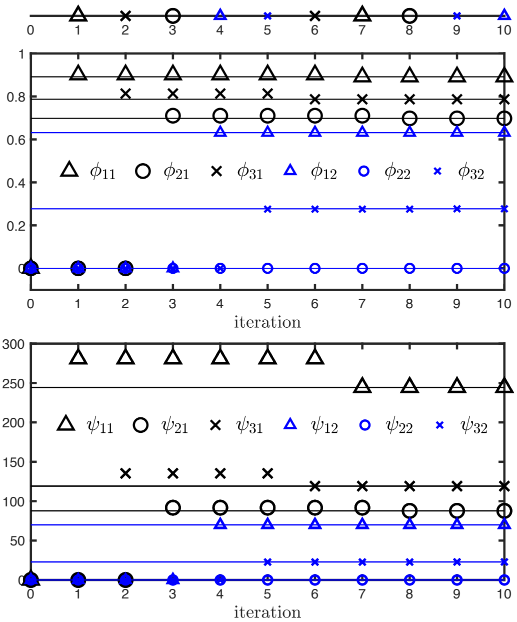

Fig. 3. Dynamics of contract profile $(\boldsymbol{\phi}, \boldsymbol{\psi})$ under an iterative process of signing optimal contracts.

Netflix's, i.e., $u_3 = 117.67$ dollars. Because the peering contracts are confidential, we estimate the (best-quality) peering costs of APs in the range of 50 to 100 of million dollars per annum and set $(v_1, v_2) = (75, 75)$ million. Because users' average throughput almost doubled after the signing of the Netflix-Comcast peering [76], we use $g = 0.5$ for the gain function $G(\phi_l)$ in our evaluations, assuming that users' choices over providers are proportional to their throughput.

Under any contract profile $(\boldsymbol{\phi}, \boldsymbol{\psi})$, we evaluate the maximum profit growth of any pair $l = (m, n)$, defined by $\Delta_l(\boldsymbol{\phi}, \boldsymbol{\psi}) \triangleq P_m(\phi_l^*(\boldsymbol{\phi}_{-l}), \boldsymbol{\phi}_{-l}, \psi_l^*(\boldsymbol{\phi}_{-l}), \boldsymbol{\psi}_{-l}) - P_m(\boldsymbol{\phi}, \boldsymbol{\psi}) + Q_n(\phi_l^*(\boldsymbol{\phi}_{-l}), \boldsymbol{\phi}_{-l}, \psi_l^*(\boldsymbol{\phi}_{-l}), \boldsymbol{\psi}_{-l}) - Q_n(\boldsymbol{\phi}, \boldsymbol{\psi})$, which is achieved under the optimal contract $(\phi_l^*(\boldsymbol{\phi}_{-l}), \psi_l^*(\boldsymbol{\phi}_{-l}))$. Since profit growth reflects the incentives for peering, we emulate an iterative process under which the pair of CP-AP that has the highest value of Δ_l will engage an optimal contract, and we start the process with a peering profile $\boldsymbol{\phi} = \mathbf{0}$ that indicates no CP-AP peering contract was established at the beginning.

Figures 3 plots the contract profiles $(\boldsymbol{\phi}, \boldsymbol{\psi})$, where the legend at the top shows the pair l that engages an optimal contract in each iteration. We observe that after 10 rounds of iterations, the outcome has converged to the contractual equilibrium, which is shown by the levels of horizontal lines. In general, the contractual transfers show similar trends as the contractual levels and a contract of higher level is associated with a higher value of transfer. Intuitively, a contract of high level implies that despite of high peering costs, the contract will bring higher revenue for the CP such that both parties are willing to absorb the cost and share high profit growth. In particular, we also observe that the first peering pair (in iteration 1) is indeed Netflix-Comcast. Figure 4 plots the corresponding market share π_m of CPs and profits P_m and Q_n of all providers. We observe that when an optimal contract increases its level ϕ_l between a pair $l = (m, n)$ at any iteration, CP m's market share π_m and profit P_m increase while those of other CPs decrease. When comparing the market share and profits under the contractual equilibrium with those at the beginning (iteration 0), we observe that all CPs obtain higher market share (as a result of a lower

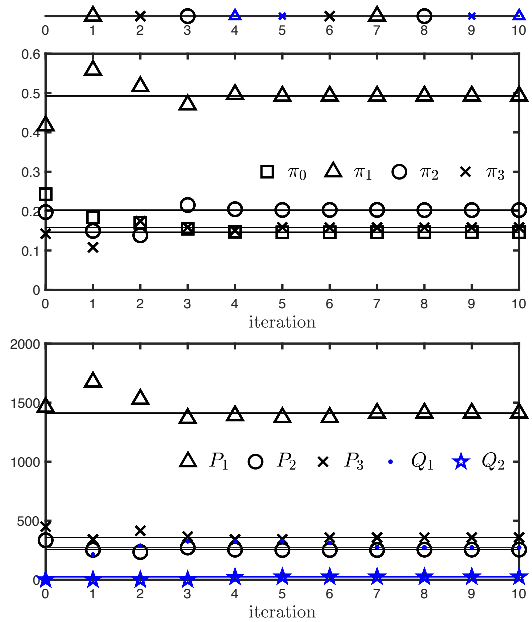

Fig. 4. Dynamics of CPs' market share π_m and providers' profit P_m and Q_n under the iterative process.

π_0 due to the increase in overall market penetration), but lower profit (due to market competition and monetary transfer to APs).

We next study whether the above observations[5] are general and try to reveal 1) what type of CP-AP pairs have strong incentives to sign the first peering contract when such a peering relationship does not exist? 2) what are the best peering strategies for any pair of CP-AP, if other providers start to engage in such peering contracts? and 3) how does a contractual equilibrium look like (or what are its structural properties?) when contracts are reached in a steady state.

4.1 The First Contract

We first study who have strong incentives to initiate peering when $\boldsymbol{\phi} = \mathbf{0}$ and try to understand why Netflix-Comcast happened to be first movers to engage in peering.

THEOREM 4.1. *If the providers of a pair $l = (m, n)$ are superior to those of another pair $k = (i, j)$, i.e., $m \succcurlyeq i$ and $n \succcurlyeq j$, and CP m is non-dominant after signing an optimal contract, i.e., $\pi_m(\phi_l^*(\mathbf{0}), \mathbf{0}) < 50\%$, the pairs l and k's optimal contracts satisfy $\phi_l^*(\mathbf{0}) \geq \phi_k^*(\mathbf{0})$ and their profit growths satisfy $\Delta_l \geq \Delta_k$.*

Theorem 4.1 shows that the optimal contractual level ϕ_l^* is positively correlated to the superiority of providers who sign the contract. Intuitively, high per-user revenue u_m and low peering cost v_n generate high profits from peering and therefore motivate providers to contract at high levels. Note that superiority by its own cannot guarantee the maximum of profit growth, although providers' weights α_m and β_n play a role in determining the market share. Theorem 4.1 shows that the maximum profit growth can be guaranteed if the CP is not dominant because the improved effective weight $\alpha_m \beta_n G(\phi_l^*)$ can attract enough additional users as its existing market share is not more than half of the entire market.

[5]Although detailed input values change the quantitative results, we find that the qualitative observations that we make remain the same.

Since Netflix and Comcast had the largest market share in 2013, their weights α_1 and β_1 are the highest among competitors. In addition, as their unit revenue u_m and cost v_n were no worse than their competitors, they are *superior* to others by Definition 3.2. Thus, Theorem 4.1 implies that the Netflix-Comcast pair has the strongest incentive to peer, although Netflix's market share exceeds 50% under our emulation because the additional number of users it can attract is still substantial. The implication of Theorem 4.1 also help us understand that with respect to any AP (CP), the first peering contract is likely to be signed with the most superior CP (AP). For example, the most superior CP Netflix is also the first one that signed a contract with Verizon [48] and the largest AP Comcast is the first AP that Netflix peered with.

4.2 Best Response of Peering Contract

After understanding how the first peering contracts were signed, we continue to study how providers will adapt their optimal contracts when other pairs of providers engage in private peering, which influences the market share, profits, and therefore, the optimal contracts of the providers.

We first consider from a CP m's perspective and analyze how its optimal contract (ϕ_l^*, ψ_l^*) at a peering link $l \in \mathcal{L}^m$ is influenced by its other contracts at peering links $k \in \mathcal{L}^m$.

Theorem 4.2 (Contract Cannibalism). *For any CP $m \in \mathcal{M}$ and one of its contract link $l \in \mathcal{L}^m$, the optimal contractual level $\phi_l^*(\boldsymbol{\phi}_{-l})$ and transfer $\psi_l^*(\boldsymbol{\phi}_{-l})$ are non-increasing in any other contractual level ϕ_k of CP m, i.e., $k \in \mathcal{L}^m$.*

Theorem 4.2 states that a CP's optimal contractual level with and the corresponding transfer to an AP is negatively correlated with the contractual level at which it peers with other APs. As a CP m strengthens its peering $l = (m, n)$ with an AP n, the weight $\alpha_m \beta_n G(\phi_l)$ increases, and due to market cannibalism, its market share component from other peering links will decrease, which decreases the necessity of maintaining a high contractual level over those links. In economics, the result of Theorem 4.2 shows that the peering decisions, i.e., contractual levels, of all the links of a CP are strategic substitutes [10] which mutually offset each other.

Next, we analyze how a CP's optimal contract (ϕ_l^*, ψ_l^*) is influenced by the peering contracts established by other CPs.

Theorem 4.3. *For any CP $m \in \mathcal{M}$ and its contract link $l \in \mathcal{L}^m$, the optimal contractual level $\phi_l^*(\boldsymbol{\phi}_{-l})$ is non-decreasing in any contractual level ϕ_k of other CPs, i.e., $k \notin \mathcal{L}^m$, if CP m is market dominant, i.e., $\pi_m(\phi_l^*(\boldsymbol{\phi}_{-l}), \boldsymbol{\phi}_{-l}) \geq 50\%$; otherwise, $\phi_l^*(\boldsymbol{\phi}_{-l})$ and $\psi_l^*(\boldsymbol{\phi}_{-l})$ are non-increasing in ϕ_k.*

Intuitively, due to market cannibalism, a CP should weaken its peering contracts if the contracts of other CPs are strengthened. Theorem 4.3 tells that this intuition does not hold when a CP dominates the market. Because a dominant CP has been chosen by the major of users, its incentive to strengthen contracts for higher market share is low. However, this incentive will increase when it loses users due to the competition from other CPs; and therefore, the dominant CP will behave aggressively to compete for the loss of market share to other CPs. The reversed argument hold for non-dominant CPs so that they will have even fewer incentives to compete for market share under competition. Theorem 4.3 also shows that the peering decisions of the links of different non-dominant CPs are negatively correlated and thus they are strategic substitutes which mutually offset each other.

Both Theorems 4.2 and 4.3 help understand our observations made from Figure 3. First, the adjustments of contractual level ϕ_l and transfer ψ_l are often in the same direction for optimal contracts, which explains the similar trends for ϕ_l and ψ_l in the figure. As a result, we will mainly focus on the analysis of contractual level ϕ_l. Second, we can explain the weakening of contracts after an initial period (iterations 1-5) of contract engagement. For example, Amazon weakens its optimal contract (signed at iteration 2) with Comcast (at iteration 6) due to the strengthened contracts from other CPs (iterations 3, 4) and itself (iteration 5). Another example is that Netflix weakens its optimal contract (signed at iteration 1) with Comcast (at iteration 7) due to the signed contracts from other CPs and itself (iteration 4).

4.3 Structure of Contractual Equilibrium

In this subsection, we study the steady-state contracts that are formed under a contractual equilibrium $(\boldsymbol{\phi}^*, \boldsymbol{\psi}^*)$, where each individual contract is optimal and each pair of CP-AP would not have incentives to modify the contract terms.

Theorem 4.4. *If a pair $l = (m, n)$ of providers are superior to those of another pair $k = (i, j)$, i.e., $m \succcurlyeq i$ and $n \succcurlyeq j$, we have $\phi^*_l \geq \phi^*_k$ under any contractual equilibrium $(\boldsymbol{\phi}^*, \boldsymbol{\psi}^*)$.*

Aligned with Theorem 4.1, Theorem 4.4 shows that contractual level ϕ^*_l under contractual equilibrium is positively correlated to the superiority of the contracting CP and AP. This explains the observations in Figure 3 that for any AP, Netflix's contractual level is higher than other CPs; and for any CP, the contractual level is higher with Comcast. After reaching a contractual equilibrium, each pair's weight increases from $\alpha_m \beta_n G(0)$ to $\alpha_m \beta_n G(\phi^*_l)$. Since peering improves service quality, it will attract users of CP 0 who chose outside options and therefore, the penetration of the entire Internet content market will increase, shown by the decreasing value of π_0 in Figure 4. This result also implies that the engagement of private peering will enlarge the ratio of market share between the market leader and follower, because the multiplier $G(\phi^*_l)$ of weights is monotonic in ϕ^*_l. This explains the observation in Figure 4 that Netflix's market share π_1 under the contractual equilibrium has increased substantially compared to those of Hulu and Amazon. Although peering is a means for superior providers with better intrinsic characteristics to enlarge their advantages in terms of market share, it does not guarantee CPs for higher profit (also seen from Figure 4) due to the contractual transfers to APs and fiercer market competition introduced by peering.

Theorem 4.4 compares the contracts of providers where the superiority relation applies, e.g., a CP's weight α_m and per-user revenue u_m are both better than those of another CP. However, for Hulu and Amazon, we have $\alpha_2 > \alpha_3$, but $u_2 < u_3$, and Theorem 4.4 does not specify whether there is a similar ordering between them, since the superiority relation only provides a partial ordering of the providers. The next result shows that such a strict ordering does exist as a structural property of a contractual equilibrium.

Corollary 4.5 (Full Ordering under Equilibrium). *Under a contractual equilibrium, there exists a full ordering of the CPs: for any two CPs $m, m' \in \mathcal{M}$ such that if $\phi^*_{mn} \geq \phi^*_{m'n}$ for some $n \in \mathcal{N}$, $\phi^*_{mn'} \geq \phi^*_{m'n'}$ holds for all $n' \in \mathcal{N}$. Similarly, such a full ordering of the APs also exists.*

Corollary 4.5 specifies that a strict ordering of the providers exists under a contractual equilibrium. This implies that when peering with Comcast, if the contractual level of Amazon is higher than that of Hulu, Amazon is superior to Hulu under the contractual equilibrium such that with any AP, its contractual level would not be lower than that of Hulu. This is indeed the case shown in Figure 3, where the most inferior pair of providers, Hulu and Verizon, do not even sign a contract, i.e., $\phi^*_{22} = 0$, under equilibrium. Notice that this ordering is specific to a contractual equilibrium. When market conditions change, the contractual equilibrium and the corresponding strict ordering may change.

To conclude this section, we find that the strategic peering behavior of providers and the structural characteristics of contractual equilibrium primarily depend on 1) the superiority of the providers and 2) whether a CP is market dominant. Superior providers tend to peer at high contractual levels and obtain high market shares. In response to the increase of contractual levels by other pairs of providers, the optimal contractual level of a CP-AP pair often decreases due to market

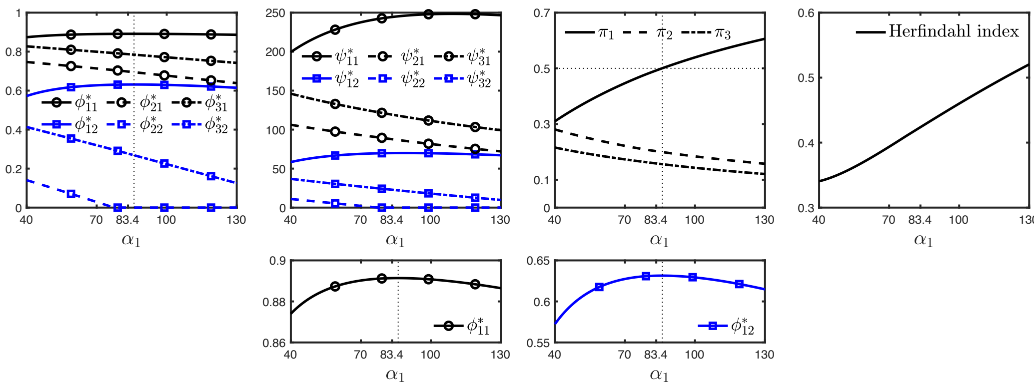

Fig. 5. Contractual equilibrium, market shares and Herfindahl index of CP market under varying α_1.

cannibalism. However, the only exception happens when the CP dominates the market and will behave oppositely.

5 EVOLUTION OF PEERING CONTRACTS

In the previous section, we analyzed the strategic peering behaviors of providers using 2013 year-end market data as model parameters and emulated an iterative process that starts without any peering contract but leads to a contractual equilibrium. This equilibrium specifies the contractual levels and transfers that would be engaged between CPs and APs. Because the equilibrium depends on the exogenous variables of the model, i.e., the intrinsic characteristics (α_m, u_m) of CP and (β_n, v_n) of AP, and market environment parameters α_0 and g, to understand the current and future peering contracts, we study the evolution of equilibrium by taking the contractual equilibrium resulted from the previous iterative process as a starting point in this section. In particular, we will focus on how peering contracts $(\boldsymbol{\phi}^*, \boldsymbol{\psi}^*)$ and CPs' market share π_m are influenced by the changing parameters (α_m, u_m) of CPs and (α_0, g) of the market environment, because such private peering contracts primarily impact the development and competition of the content market. We will measure the competitiveness of the CP market using the Herfindahl index [34] defined by $\sum_{m=1}^{M}(\pi_m)^2/(\sum_{m=1}^{M}\pi_m)^2$. An increase in the Herfindahl index generally indicates a decrease in competition.

5.1 Impact of CPs' Characteristics

In this subsection, we study how a CP's weight α_m and per-user revenue u_m influence the contractual equilibrium. As new contents have been produced and only available on Internet-based platforms, users have become more attracted to certain CPs. For example, Netflix's market share has continuously increased from 38% in 2013 [63] to 64.5% in 2016 [69]. This implies that CP's weight α_m has been increasing over recent years. In Figure 5, we vary the weight of Netflix, i.e., α_1, along with the x-axis and plot the contractual equilibrium and the corresponding market share and Herfindahl index of the CP market. The initial equilibrium we obtained from the previous section can be referred to as the point at $\alpha_1 = 83.4$ shown on the x-axis. We observe that as Netflix's weight increases, its market share π_1 increases, but that of Hulu or Amazon decreases. This leads to a less competitive CP market whose Herfindahl index increases. We also observe that as α_1 increases, the contractual level and transfer of contracts engaged by Hulu and Amazon always decrease; however, those of contracts engaged by Netflix show a single peak pattern, better observed from the lower

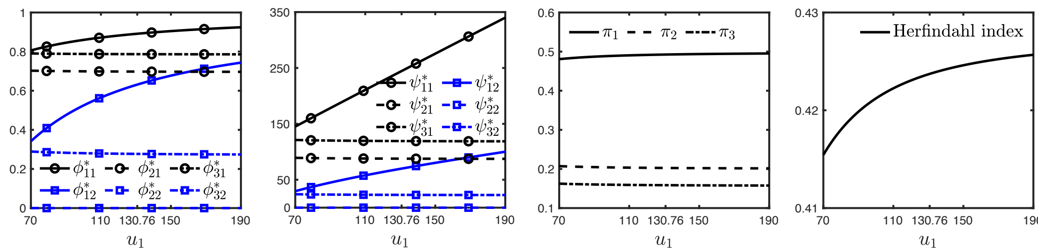

Fig. 6. Contractual equilibrium, market shares and Herfindahl index of CP market under varying u_1.

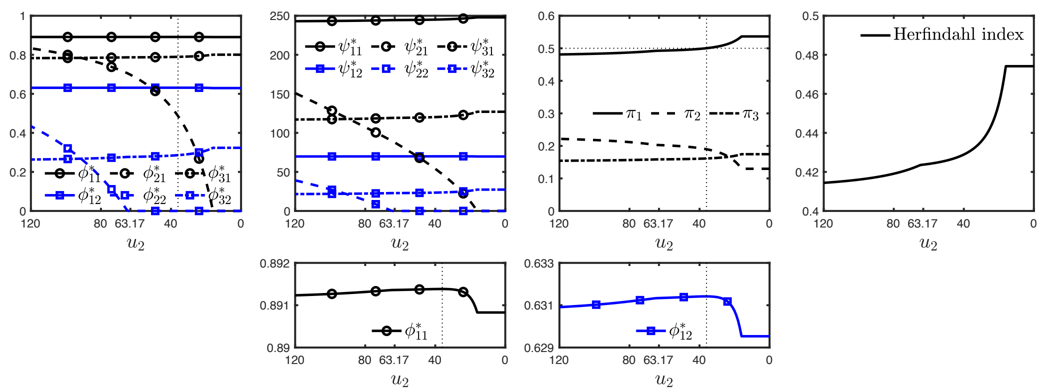

Fig. 7. Contractual equilibrium, market shares and Herfindahl index of CP market under varying u_2.

subfigures. In particular, Netflix's contracts with APs strengthen (weaken) as α_1 increases before (after) it becomes market dominant.

Meanwhile, Netflix will increase its price from \$8 to \$9 per month in 2019 [53], which implies that its per-user revenue will increase. In Figure 6, we vary Netflix's per-user revenue u_1 along the x-axis and plot the same metrics as in Figure 5. The initial equilibrium can be referred to as the point at $u_1 = 130.76$. We observe that as u_1 increases, similar to Figure 5, π_1 increases, while π_2 and π_3 decrease, leading to a less competitive CP market with a higher Herfindahl index. Although the change in market share is not as substantial as that in Figure 5, the monetary transfer from Netflix to the APs increases linearly with u_1. In contrast to the case in Figure 5, the contractual level and transfer of contracts engaged by Netflix increase monotonically with its per-user revenue u_1, while those of contracts engaged by Hulu and Amazon decrease mildly with u_1.

In contrast to Netflix, Hulu is expected to decrease its price in 2019 from \$7.99 to \$5.99 per month [30], which implies that its per-user revenue will decrease. In Figure 7, we vary Hulu's per-user revenue u_2 along the x-axis and plot the same metrics as before. The initial equilibrium can be referred to as the point at $u_2 = 63.17$. We observe that as u_2 decreases, π_2 decreases, while π_1 and π_3 increase, again leading to a less competitive CP market with a higher Herfindahl index. We also observe that as u_2 decreases, Amazon's contracts with APs strengthen mildly, while the contractual level and transfer of contracts engaged by Hulu drop prominently. If Hulu's per-user revenue drops below $u_2 = 16$, it will not sign any contract, and the equilibrium remains afterward. Similar to the case in Figure 5, we observe that the contractual level and transfer of contracts engaged by Netflix show a single peak pattern (better observed from the lower subfigures), which strengthen (weaken) as u_2 decreases before (after) it becomes market dominant.

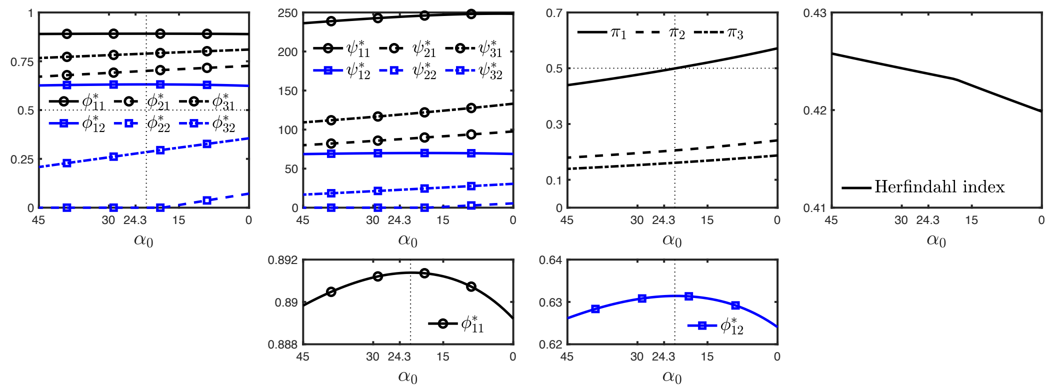

Fig. 8. Contractual equilibrium, market shares and Herfindahl index of CP market under varying α_0.

In summary, given the changes in CPs' characteristics, we find that the CP market is becoming less competitive. We foresee that most peering contracts will not strengthen, because even Netflix's incentives for peering will decrease after it becomes market dominant and continues to grow in market share. After making observations and conclusions from the above evaluations, we next study whether these observations and conclusions hold in general.

Corollary 5.1. *If any CP m's intrinsic parameter α_m or u_m increases, its market share π_m under contractual equilibrium will increase, while that of other CPs will decrease.*

Corollary 5.1 confirms our observations on the monotonic changes in market shares. Intuitively, it states that if any CP becomes superior, its market share will increase while that of the others will decrease.

Theorem 5.2. *For any non-dominant CP m and its peering link $l \in \mathcal{L}^m$, the equilibrium contractual level ϕ_l^* will increase if its intrinsic parameter α_m or u_m increases, and will decrease if any other CP i's intrinsic parameter α_i or u_i increases.*

Theorem 5.2 confirms our observations on the peering behavior of a non-dominant CP. It states that a non-dominant CP will engage in peering more aggressively to obtain market share if it becomes superior; however, it will weaken their contracts due to market cannibalism, if any of the other CPs, whether dominant or not, becomes superior. This behavior of non-dominant CPs is aligned with the off-equilibrium best-response results of Theorems 4.2 and 4.3.

Theorem 5.3. *For any dominant CP m and its peering link $l \in \mathcal{L}^m$, the equilibrium contractual level ϕ_l^* will increase if its per-user revenue u_m or any other CP i's intrinsic parameter α_i or u_i increases, and will decrease if its weight α_m increases.*

Theorem 5.3 states the non-intuitive peering behavior of the dominant CP that also confirms with our observations. Although the CP will still enhance its peering contracts as its per-user revenue increases, it will behave oppositely as non-dominant CPs otherwise. First, as its weight α_m further increases, because it has already obtained the majority of market share, its incentive to peer will decrease. Second, as other CPs become more superior and engage in peering more aggressively, the dominant CP will compete for the loss of market share by enhancing its peering contracts in response. This particular behavior of the dominant CP is aligned with the off-equilibrium best-response result of Theorem 4.3. The implication of Theorem 5.2 and 5.3 tell that as Netflix becomes dominant and superior, most peering contracts, even Netflix's, will weaken. This helps explain why peering disputes did not occur recently.

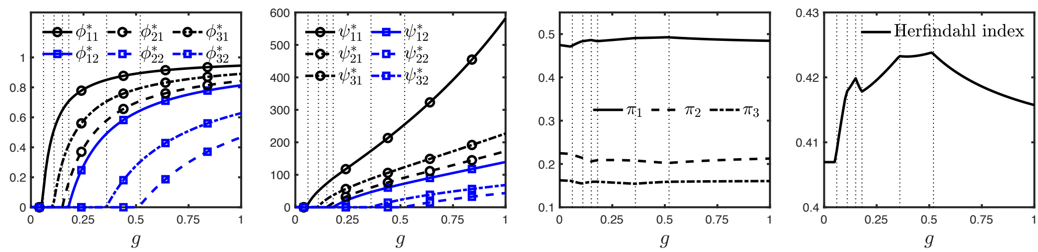

Fig. 9. Contractual equilibrium, market shares and Herfindahl index of CP market under varying g.

5.2 Impact of Market Environments

After understanding how the contractual equilibrium is influenced by CPs' characteristics, in this subsection, we study how it is influenced by the market parameters α_0 and g.

As the digital video penetration has been continuously increasing and is projected to reach over 83% by 2020 [68], we foresee that the attractiveness α_0 of outside options such as TV and DVDs to users will decrease. In Figure 8, we vary the weight α_0 of the conceptual CP 0 of outside options along the x-axis and plot the same metrics as before. The initial equilibrium can be referred to as the point at $\alpha_0 = 24.3$. We observe that as α_0 decreases, all real CPs' market shares increase, leading to a more competitive CP market with a lower Herfindahl index. Similar to the case in Figure 7, we observe that although the contractual level and transfer of contracts engaged increase in general as α_0 decreases, Netflix's contracts weaken after it becomes dominant, resulting in a faster decline in the Herfindahl index as α_0 further decreases.

THEOREM 5.4. *When the Internet content penetration increases, i.e., α_0 decreases, any real CP m's market share π_m will increase. Furthermore, for any CP m's peering link $l \in \mathcal{L}^m$, ϕ^*_l will decrease if CP m is dominant; otherwise, ϕ^*_l will increase.*

Theorem 5.4 confirms our observations on the monotonic changes in CPs' market shares, which is due to the redistribution of the additional market shares lost by outside options. As the overall market penetration increases, the total number of users of CPs also increases, resulting in higher incentives for non-dominant CPs to engage in private peering to attract more users. Nonetheless, the opposite behavior of the dominant CP is also observed here, i.e., it will weaken its contracts as its market share exceeds 50%. Combined with Theorem 4.1, this result helps to understand that the increase in market penetration also contributes to the effect of increasing the weight α_m of CPs, resulting in superior CPs and higher incentives to switch from public to private peering.

After studying impact of market penetration, we consider the impact of parameter g in the gain function $G(\phi_l) = 1 - g(1 - \phi_l)^2$. When g increases, the discount on the weight $\alpha_m \beta_n$ will be more prominent, reducing the attractiveness of pair of providers more quickly. This can be used to model the scenario where users are becoming more sensitive to service quality. With the emergence of new real-time multimedia applications, e.g., 3D virtual reality, as well as new standards, e.g., superHD [62], that require higher video resolutions, we expect that users will be more sensitive to service quality and cannot be satisfied with low levels of peering. This implies that the value of g will increase for the near future. In Figure 9, we vary the sensitivity parameter g of the gain function along the x-axis and plot the same metrics as before. The initial equilibrium can be referred to as the point at $g = 0.5$. We observe that if g is very small, i.e., users are not sensitive to quality, CPs would not have incentives to peer. As g increases, providers start to engage in peering and strengthen their contracts, i.e., increasing both the contractual level ϕ^*_l and transfer ψ^*_l. We observe that the sequence and intensity of peering contracts follow the strict ordering of superiority, i.e., Netflix > Amazon > Hulu, characterized in Corollary 4.5. Although the plots for market share and Herfindahl index show some non-smooth turning points, they occur when a CP-AP pair switches from public peering, i.e., $\phi_l^* = 0$, to private peering. Before any peering contract is engaged, we observe that the market share of each CP decreases as g increases, due to the loss of market share to outside options. Since the multiplier $G(0)$ is the same for all CPs, their relative market shares remain the same, so as the Herfindahl index. From the bottom right subfigure, we observe that after Netflix signs the first contract with Comcast, the market will become less competitive as g increases. This is because Netflix is the only CP that provides a quality service in the market and its advantage becomes more prominent when users are more sensitive to quality. However, as Amazon and Hulu start to peer with Comcast, this increasing trend in the Herfindahl index gets dampen and even reverted, respectively. This pattern continues as Netflix start to peer with Verizon, followed by Amazon and Hulu.

To conclude this section, we find that the optimal peering strategies of providers and the evolution of contractual equilibrium are also directly impacted by the superiority and market dominance of providers. In particular, any non-dominant CP will enhance its peering contracts when it becomes superior and will weaken them when another CP becomes superior. The dominant CP, however, will behave oppositely, except that it will also raise the contractual levels when its per-user revenue increases. With the growing market penetration, the contractual levels of non-dominant CPs will increase, while that of the dominant CP will decrease, resulting in a fiercer competition in the content market.

6 RELATED WORK

Evolution of Internet peering: The pioneering work of Gao [28] analyzed the Border Gateway Protocol (BGP) and used BGP routing data to infer and validate the peering relationships between autonomous systems (ASes). Because peering forms the Internet topology, a voluminous literature studied the impact of peering on the evolution of the Internet topology. Chang et al. [11] and Oliveira et al. [60] studied the evolution of the AS-level topology based on a decision model of peering and an empirical model, respectively. Dhamdhere et al. [21] proposed a value-based quantitative framework to study peering agreements. Jesus et al. [40] studied the topological properties induced by cascading interconnection agreements. Lodhi et al. [45] and Dhamdhere et al. [20] used agent-based modeling and simulation to study the network formation process and the peering decision of transit ISPs, respectively. Tan et al. [72] investigated peering arrangements at the Internet backbone intending to identify optimal peering strategies which help backbone providers improve routing decisions and service quality. As the Internet was primarily utilized as a communication network, prior works focused on the *transit service* sold by ASes via peering and the corresponding two traditional forms of peering: provider-to-customer and peer-to-peer, which were conjectured to be the only stable peering models [39]. With the rise of video streaming giant Netflix recently, the Internet has been evolving into a content-centric network. Based on traceroute measurements, Gill et al. [29] revealed a flattening phenomenon of the Internet topology, while Dhamdhere et al. [20] explored the explanations for such a topological transition towards a flatter mesh. Through measurements of inter-domain traffic during 2007-2009, Labovitz et al. [43] identified significant changes in inter-AS traffic patterns and evolution of ASes' peering strategies. Based on coalition game theory, Ma et al. [50] showed that a reverse provider-to-customer peering would emerge. Faratin et al. [24] and Lodhi et al. [46] analyzed the complexity of peering and discussed the emergence of paid peering (also known as premium peering).

This new form of peering is driven by the new requirements from delay/throughput sensitive CPs that value service quality more than best-effort connectivity, while existing forms of peering were unable to address. For example, Netflix uses multiple third-part content delivery networks (CDNs), e.g., Akamai, Level 3 and Limelight [4], to deliver its content to end users. In turn, CDNs collaborate, i.e., establish paid peering, with ISPs (including APs) for hosting their servers in ISPs' data centers. A few work explored challenges and proposed solutions in CDN-ISP collaborations [25, 33, 44]. Frank et al. [25] identified two key enablers, i.e., informed end-user to server assignment and in-network server allocation, based on which they designed a prototype system for supporting the collaborations and improving content delivery performance. Herbaut et al. [33] proposed a CDN as-a-Virtual-Network-Function approach to tackle the Server Selection (SS) ones and Traffic Engineering (TE) problems [41] in the collaborations. Lai et al. [44] designed and implemented an architecture using SDN and NFV techniques to enhance request rerouting in the collaborations for improving user-perceived QoS. In recent years, many ISPs, especially large APs, e.g., Comcast and Verizon, have built their own CDNs [2, 3]. Plus end-users still rely on the last-mile APs for accessing the Internet and obtaining CPs' contents, CPs have been increasingly established paid peering with APs, e.g., Netflix-Comcast [76] and Netflix-Verizon [31]. In this work, we study the peering contracts, a type of collaboration between CPs and APs. In particular, we explore the formation and evolution of CP-AP collaborations from an economic perspective, while the prior work on CDN-ISP collaborations mainly focused on designing new systems, architectures or approaches for supporting the collaborations and improving user-perceived QoS from a technical perspective.

Modeling of paid peering: Some recent work modeled and analyzed paid peering between CPs and APs from an economic perspective. Gyarmati et al. [32] and Courcoubetis et al. [16] proposed a churn model to determine the fair prices of paid peering between CPs and APs. Zarchy et al. [77] designed a techno-economic interconnection framework to address the various types of peering disputes. All the three work used the concept of Nash bargaining [55]. Ma [49] studied CPs' strategic peering decisions given the paid peering options provided by APs. Courcoubetis et al. [17] made a qualitative study on the impacts of paid peering agreements on social welfare and providers' surplus. Wang et al. [74] explored the optimal peering scheme for an AP to offer CPs one of or both paid peering or settlement-free peering under profit-optimal or welfare-optimal objective.

The prior work except Ma's work [49] did not model the interactions among peering contracts engaged by different CP-AP pairs, i.e., they did not capture the competition among multiple providers (neither CPs nor APs). Ma's work [49] only analyzed the impact of competition on paid peering in the markets with monopoly or duopoly CPs or APs. Compared to them, our work is the first to model and analyze the interactions among different CP-AP pairs' peering contracts in the markets including any number of CPs and APs. Our general model and analysis can provide a more universal understanding on the formation and evolution of peering contracts under different Internet market structures including monopoly, duopoly, and oligopoly which all widely exist in many countries and regions. The prior work all modeled the contract term only by a monetary transfer under which providers can only negotiate transfer prices if they sign contracts. Compared to the prior work, we model the contract term by a tuple of monetary transfer and contractual level under which providers can negotiate not only the transfer prices but also the degrees of contracts, i.e., the quality of private peering. This generalization provide a more faithful characterization on the real cases that APs can deploy different amount of resources, e.g., bandwidth or cache, to build different qualities of private peering based on which CPs pay different prices to APs. Besides, our work first reveal that market dominance plays a key role on the peering contract strategies of providers. This new finding shows the difference of formation and evolution of peering contracts in highly competitive markets and monopolized markets, which could help regulatory authorities to understand private peering contracts and legislate desirable regulations.

Network games and club goods: The model in our work can be considered as a network game[6] on a bipartite graph of CPs and APs. Network games and their equilibria have been studied in some literature of economics [5, 27, 54, 61]. In their models, the vertices in networks represent players who individually decide their strategies and an edge exists between two vertices if a vertex's payoff is influenced by the other's strategy. Unlike them, our model is a physical model where vertices are the physical entries of CPs and APs who jointly decided contract strategies and edges represents the peering links between CPs and APs[7]. Besides, for tractability, the literature usually assumes that the utility functions of players are bilinear functions (e.g., [5]), linear-quadratic functions (e.g., [54]), or functions of the linear combination of other players (e.g., [27, 61]). In our work, the utility functions of providers are constructed based on a contract model and a user choice model, and does not satisfy any of the assumptions in the literature.

In our scenario, the content services provided by CPs can be considered as club goods. All the end-users subscribing to a CP's service form a group/club and each user is a member of the club. Theory of club goods has been studied in some literature of economics [6, 9, 15, 59, 66, 70]. Most of the literature (e.g., [6, 9, 15, 59]) studied the resulting outcome under the hypothesis of coordinated actions by a club's members to optimize the welfare of the club, while our work focused on the non-cooperative peering actions by the providers to optimize their individual profits. A few literature (e.g., [66, 70]) examined Nash equilibrium in club games, where the providers choose strategies, e.g., provision levels or membership prices, to maximize their profits. When using their model to capture our problem, the Nash equilibrium generated is the equilibrium under group deviation mentioned in Section 2, which is sufficient of being our contractual equilibrium.

"Nash-in-Nash" solution and bilateral oligopoly: From a modeling perspective, our characterization of the steady-state peering contracts, i.e., *contractual equilibrium*, constitute a "Nash-in-Nash" solution, i.e., separate bilateral Nash bargaining problems within a Nash equilibrium to a game played among all pairs of firms, which has been explored and employed in some literature of industrial organization [8, 14, 22, 37]. Collard-Wexler [14] proposed a non-cooperative foundation for the "Nash-in-Nash" solution between multiple upstream and downstream firms, which provides strong support for the solution as a viable surplus division rule. Based on "Nash-in-Nash" solution, Dobson and Waterson [22] examined the competition and welfare effects of vertical price fixing through industry-wide resale price maintenance arrangements. Björnerstedt and Stennek [8] developed a model where upstream and downstream firms meet to negotiate contracts specifying prices and quantities in simultaneous and interdependent Rubinstein-Ståhl negotiations. Horn and Wolinsky [37] analyzed a duopoly in which firms acquire inputs through bilateral monopoly relations with suppliers to explore how input prices and profits are affected by the structures of the upstream and downstream industries. [22, 37] focused on the cases where the upstream or downstream markets include monopoly or duopoly firms, while [8, 14] and our work consider the more general upstream or downstream markets including oligopoly firms. Compared to [8, 14], our work only not captured the structure of "Nash-in-Nash" solution, but also studied its sensitivity to varying market parameters, which helps understand the evolution of "Nash-in-Nash" solution in fast-developing Internet markets.

[6]In a broad sense, network games (also known as games on network) are the games which capture the strategic interactions of individual players in structured networks/graphs.

[7]Notice that if we model CPs and APs as the players in the network game framework, an edge might exist for a CP-AP pair even they do not have a direct peering link, because their strategies on other links might affect each other's aggregate utility. On the other hand, if we take each CP-AP pair as a player in the network game framework, the formed network would be a complete graph where a vertex is a CP-AP pair and any two vertices are connected by an edge, because the utility of a CP-AP pair is influenced by any other CP-AP pair. In either case, the network does not have a special structure and the solution concept reduces to a standard Nash equilibrium for a non-cooperative game.

In our work, the Internet markets consisting of multiple CPs and APs considered is a bilateral oligopoly [26]. A body of literature in economics has examined the effects of competition, bargaining, bundling, or integration among agents/firms within traditional bilateral oligopoly environments such as health care markets [35, 36] and cable television markets [18, 19]. Similar to the prior work on modeling of paid peering [16, 17, 32, 49, 74, 77], the work [18, 19, 35, 36] also modeled the contract term only by a monetary transfer, while our work extends it to a tuple of contractual level and monetary transfer, which characterizes the cases that downstream firms can choose different amounts of resources from upstream firms and pay different prices to them.

7 CONCLUSIONS

In this paper, we study the private peering contracts between content providers (CPs) and access providers (APs). We propose a novel model which characterizes the contract terms between CPs and APs in terms of contractual levels and monetary transfers. We introduce the concept of contractual equilibrium that captures the steady-state peering contracts on the entire Internet content market. Based on our model and by using market data, we emulate the strategic peering behaviors between providers and analyze the formation and evolution of their peering contracts. The emulation and analytical results show that superiority and market dominance of providers are the two major factors in determining their optimal peering strategies. In particular, superior providers have strong incentives to start the initial peering contract and tend to peer at high contractual levels. Dominant and non-dominant CPs often have opposite optimal peering strategies as the market environments or CPs' intrinsic properties change.

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A PROOFS OF THEORETICAL RESULTS

In this appendix, we give the proofs of theoretical results.

Proof of Theorem 2.2: By substituting the expressions (1) of providers' profits into the two conditions in Definition 2.1, we know that given the contract profile $(\boldsymbol{\phi}_{-l}, \boldsymbol{\psi}_{-l})$ of all other pairs, (ϕ_l^*, ψ_l^*) is an optimal contract of the CP-AP pair l if and only if it satisfies that

$$\begin{cases} \phi_l^* \in \arg\max_{\bar{\phi}_l \in [0,1]} \left\{ U_m(\bar{\phi}_l; \boldsymbol{\phi}_{-l}) - V_n(\bar{\phi}_l; \boldsymbol{\phi}_{-l}) \right\} \\ \psi_l^* = \dfrac{1}{2} \left[U_m(\phi_l^*, \boldsymbol{\phi}_{-l}) - U_m(0, \boldsymbol{\phi}_{-l}) + V_n(\phi_l^*, \boldsymbol{\phi}_{-l}) - V_n(0, \boldsymbol{\phi}_{-l}) \right]. \end{cases}$$

Furthermore, if the function $W_l(\boldsymbol{\phi}) \triangleq U_m(\boldsymbol{\phi}) - V_n(\boldsymbol{\phi})$ is strictly concave in the level ϕ_l, it must have a unique maximum point, i.e., $\arg\max_{\bar{\phi}_l \in [0,1]} \left\{ U_m(\bar{\phi}_l; \boldsymbol{\phi}_{-l}) - V_n(\bar{\phi}_l; \boldsymbol{\phi}_{-l}) \right\}$ is a single-valued function of the levels $\boldsymbol{\phi}_{-l}$. Therefore, there exists a unique optimal contract (ϕ_l^*, ψ_l^*) and its level satisfies $\phi_l^* = \arg\max_{\bar{\phi}_l \in [0,1]} \left\{ U_m(\bar{\phi}_l; \boldsymbol{\phi}_{-l}) - V_n(\bar{\phi}_l; \boldsymbol{\phi}_{-l}) \right\}$. □

Proof of Corollary 2.4: By Definition 2.3, if $(\boldsymbol{\phi}, \boldsymbol{\psi})$ is a contractual equilibrium, the contract (ϕ_l, ψ_l) of any pair is its optimal contract, from which we can derive that for any $(\bar{\phi}_l, \bar{\psi}_l)$ satisfying $P_m(\bar{\phi}_l, \boldsymbol{\phi}_{-l}, \bar{\psi}_l, \boldsymbol{\psi}_{-l}) \geq P_m(0, \boldsymbol{\phi}_{-l}, 0, \boldsymbol{\psi}_{-l})$ and $Q_n(\bar{\phi}_l, \boldsymbol{\phi}_{-l}, \bar{\psi}_l, \boldsymbol{\psi}_{-l}) \geq Q_n(0, \boldsymbol{\phi}_{-l}, 0, \boldsymbol{\psi}_{-l})$,

$$\begin{aligned} & [P_m(\boldsymbol{\phi}, \boldsymbol{\psi}) - P_m(0, \boldsymbol{\phi}_{-l}, 0, \boldsymbol{\psi}_{-l})] \, [Q_n(\boldsymbol{\phi}, \boldsymbol{\psi}) - Q_n(0, \boldsymbol{\phi}_{-l}, 0, \boldsymbol{\psi}_{-l})] \\ = & \left[\frac{P_m(\boldsymbol{\phi}, \boldsymbol{\psi}) - P_m(0, \boldsymbol{\phi}_{-l}, 0, \boldsymbol{\psi}_{-l}) + Q_n(\boldsymbol{\phi}, \boldsymbol{\psi}) - Q_n(0, \boldsymbol{\phi}_{-l}, 0, \boldsymbol{\psi}_{-l})}{2} \right]^2 \\ \geq & \left[\frac{P_m(\bar{\phi}_l, \boldsymbol{\phi}_{-l}, \bar{\psi}_l, \boldsymbol{\psi}_{-l}) - P_m(0, \boldsymbol{\phi}_{-l}, 0, \boldsymbol{\psi}_{-l}) + Q_n(\bar{\phi}_l, \boldsymbol{\phi}_{-l}, \bar{\psi}_l, \boldsymbol{\psi}_{-l}) - Q_n(0, \boldsymbol{\phi}_{-l}, 0, \boldsymbol{\psi}_{-l})}{2} \right]^2 \\ \geq & \left[P_m(\bar{\phi}_l, \boldsymbol{\phi}_{-l}, \bar{\psi}_l, \boldsymbol{\psi}_{-l}) - P_m(0, \boldsymbol{\phi}_{-l}, 0, \boldsymbol{\psi}_{-l}) \right] \left[Q_n(\bar{\phi}_l, \boldsymbol{\phi}_{-l}, \bar{\psi}_l, \boldsymbol{\psi}_{-l}) - Q_n(0, \boldsymbol{\phi}_{-l}, 0, \boldsymbol{\psi}_{-l}) \right] \end{aligned}$$

where the equality is implied by the second condition in Definition 2.1, and the two inequalities are implied by the first condition in Definition 2.1 and the inequality of arithmetic and geometric means. Thus the contract (ϕ_l, ψ_l) is a Nash bargaining solution between CP m and AP n, and therefore, the contractual equilibrium $(\boldsymbol{\phi}, \boldsymbol{\psi})$ is a Nash-in-Nash solution. □

Before proving the remaining theorems and corollaries, we give two lemmas as a preliminary. We consider a strategic game among the CP-AP pairs in the set $\mathcal{L}$. The strategy and payoff function of each pair $l = (m, n) \in \mathcal{L}$ are the contractual level ϕ_l and the utility $W_l(\boldsymbol{\phi}) \triangleq U_m(\boldsymbol{\phi}) - V_n(\boldsymbol{\phi})$, respectively. By Karush-Kuhn-Tucker (KKT) conditions and Equation (3) and (4), we can readily deduce the following lemma.

Lemma A.1. *For any pair $l = (m, n) \in \mathcal{L}$, given the contract profile $(\boldsymbol{\phi}_{-l}, \boldsymbol{\psi}_{-l})$ of other pairs, its optimal contract (ϕ_l^*, ψ_l^*) satisfies*

$$\frac{\partial W_l(\phi_l^*, \boldsymbol{\phi}_{-l})}{\partial \phi_l} = u_m \frac{\alpha_m \beta_n G'(\phi_l^*)[1 - \pi_m(\phi_l^*, \boldsymbol{\phi}_{-l})]X}{\alpha_m \beta_n G(\phi_l^*) + \sum_{k=(i,j)\in\mathcal{L}\setminus\{l\}} \alpha_i \beta_j G(\phi_k)} - v_n \begin{cases} \leq 0 & \text{if} \quad \phi_l^* = 0; \\ = 0 & \text{if} \quad \phi_l^* \in (0,1); \\ \geq 0 & \text{if} \quad \phi_l^* = 1. \end{cases} \tag{5}$$

We define that the vector of contractual levels $\boldsymbol{\phi}$ is a *Nash equilibrium* if and only if for any pair l, it satisfies $W_l(\boldsymbol{\phi}) \geq W_l(\bar{\phi}, \boldsymbol{\phi}_{-l}), \forall \bar{\phi} \in [0, 1]$, i.e., any pair l cannot improve its utility W_l by unilaterally changing its contractual level. By Theorem 2.2 and Definition 2.3, we have the next lemma directly.

Lemma A.2. *A contract profile $(\boldsymbol{\phi}, \boldsymbol{\psi})$ is a contractual equilibrium iff $\boldsymbol{\phi}$ is a Nash equilibrium and the transfer of each pair $l = (m, n) \in \mathcal{L}$ satisfies $\psi_l = [U_m(\boldsymbol{\phi}) - U_m(0, \boldsymbol{\phi}_{-l}) + V_n(\boldsymbol{\phi}) - V_n(0, \boldsymbol{\phi}_{-l})]/2$.*

Proof of Theorem 2.5: By Lemma A.2, the existence and uniqueness of contractual equilibrium is equivalent to the existence and uniqueness of Nash equilibrium, respectively. By Debreu-Glicksberg-Fan Theorem, if $W_l(\boldsymbol{\phi})$ is quasi-concave in ϕ_l for any pair $l \in \mathcal{L}$, there must exist a Nash equilibrium on the compact and convex set $[0, 1]^{|\mathcal{L}|}$.

Next, we prove the uniqueness by contradiction. The condition (2) guarantees that $W_l(\boldsymbol{\phi})$ is concave in ϕ_l for any $\boldsymbol{\phi}_{-l}$. Suppose there exists two distinct Nash equilibria $\hat{\boldsymbol{\phi}}$ and $\tilde{\boldsymbol{\phi}}$. By concavity of $W_l(\boldsymbol{\phi})$ in ϕ_l and the maximum principle, for any pair $l \in \mathcal{L}$ and any contractual level $\bar{\phi}_l \in [0, 1]$,

$$(\bar{\phi}_l - \hat{\phi}_l)\frac{W_l(\hat{\boldsymbol{\phi}})}{\partial \phi_l} \leq 0 \quad \text{and} \quad (\bar{\phi}_l - \tilde{\phi}_l)\frac{W_l(\tilde{\boldsymbol{\phi}})}{\partial \phi_l} \leq 0.$$

By substituting $\bar{\boldsymbol{\phi}} = \tilde{\boldsymbol{\phi}}$ in the first inequality and $\bar{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$ in the second inequality, we have for any $l \in \mathcal{L}$,

$$(\tilde{\phi}_l - \hat{\phi}_l)\frac{W_l(\hat{\boldsymbol{\phi}})}{\partial \phi_l} \leq 0 \quad \text{and} \quad (\hat{\phi}_l - \tilde{\phi}_l)\frac{W_l(\tilde{\boldsymbol{\phi}})}{\partial \phi_l} \leq 0.$$

By adding the above inequalities, we further deduce for any $l \in \mathcal{L}$,

$$(\tilde{\phi}_l - \hat{\phi}_l)\left[\frac{W_l(\tilde{\boldsymbol{\phi}})}{\partial \phi_l} - \frac{W_l(\hat{\boldsymbol{\phi}})}{\partial \phi_l}\right] \geq 0$$

which is contradictory with the condition (2). □

Proof of Corollary 3.1: If the gain function $G(\cdot)$ is strictly concave, i.e., $G''(\phi_l) < 0$, by Equation (3) and (4), we can deduce for any $l = (m, n) \in \mathcal{L}$,

$$\frac{\partial^2 W_l(\boldsymbol{\phi})}{\partial \phi_l^2} = u_m \pi_l(\boldsymbol{\phi})[1 - \pi_m(\boldsymbol{\phi})]X\left[\frac{G''(\phi_l)}{G(\phi_l)} - 2\pi_l(\boldsymbol{\phi})\left(\frac{G'(\phi_l)}{G(\phi_l)}\right)^2\right] < 0 \tag{6}$$

implying that $W_l(\boldsymbol{\phi})$ is strictly concave in the contractual level ϕ_l. By Theorem 2.2 and 2.5, the uniqueness of optimal contract and the existence of contractual equilibrium are guaranteed.

For any distinct levels $\boldsymbol{\phi}^0 \neq \boldsymbol{\phi}^1$, we assume $\sum_{k=(i,j)\in\mathcal{L}\backslash\mathcal{L}_0} \alpha_i\beta_j G(\phi_k^0) \geq \sum_{k=(i,j)\in\mathcal{L}\backslash\mathcal{L}_0} \alpha_i\beta_j G(\phi_k^1)$ without loss of generality, from which

$$\sum_{m\in\mathcal{M}\backslash\{0\}} \pi_m(\boldsymbol{\phi}^0) = \frac{\sum_{k=(i,j)\in\mathcal{L}\backslash\mathcal{L}_0} \alpha_i\beta_j G(\phi_k^0)}{\sum_{k=(i,j)\in\mathcal{L}} \alpha_i\beta_j G(\phi_k^0)} \geq \frac{\sum_{k=(i,j)\in\mathcal{L}\backslash\mathcal{L}_0} \alpha_i\beta_j G(\phi_k^1)}{\sum_{k=(i,j)\in\mathcal{L}} \alpha_i\beta_j G(\phi_k^1)} = \sum_{m\in\mathcal{M}\backslash\{0\}} \pi_m(\boldsymbol{\phi}^1).$$

Thus, there are only two cases: 1) the system includes a CP $s \in \mathcal{M}\backslash\{0\}$ satisfying $\pi_s(\boldsymbol{\phi}^0) > \pi_s(\boldsymbol{\phi}^1)$, and 2) each CP $m \in \mathcal{M}\backslash\{0\}$ satisfies $\pi_m(\boldsymbol{\phi}^0) = \pi_m(\boldsymbol{\phi}^1)$. For the first case, because

$$\frac{\sum_{k=(s,j)\in\mathcal{L}_s} \alpha_s\beta_j G(\phi_k^0)}{\sum_{k=(i,j)\in\mathcal{L}} \alpha_i\beta_j G(\phi_k^0)} = \pi_s(\boldsymbol{\phi}^0) > \pi_s(\boldsymbol{\phi}^1) = \frac{\sum_{k=(s,j)\in\mathcal{L}_s} \alpha_s\beta_j G(\phi_k^1)}{\sum_{k=(i,j)\in\mathcal{L}} \alpha_i\beta_j G(\phi_k^1)},$$

there must exist a pair $r = (s,t) \in \mathcal{L}_s$ including the CP s satisfying $G(\phi_r^0) > G(\phi_r^1)$. For the second case, we can also find a pair $r = (s,t) \in \mathcal{L}_s$ satisfying $G(\phi_r^0) > G(\phi_r^1)$; otherwise, the contractual levels $\boldsymbol{\phi}^0$ and $\boldsymbol{\phi}^1$ must be the same which is contradictory with our assumption. Because the gain function G is increasing and strictly concave in the level ϕ, we know that $\phi_r^0 > \phi_r^1$ and $G'(\phi_r^0) < G'(\phi_r^1)$. Because

$$(\phi_r^0-\phi_r^1)\left[\frac{W_r(\boldsymbol{\phi}^0)}{\partial\phi_r} - \frac{W_r(\boldsymbol{\phi}^1)}{\partial\phi_r}\right] = (\phi_r^0-\phi_r^1)u_s X\left[\frac{\alpha_s\beta_t G'(\phi_r^0)\left(1-\pi_s(\boldsymbol{\phi}^0)\right)}{\sum_{k=(i,j)\in\mathcal{L}} \alpha_i\beta_j G(\phi_k^0)} - \frac{\alpha_s\beta_t G'(\phi_r^1)\left(1-\pi_s(\boldsymbol{\phi}^1)\right)}{\sum_{k=(i,j)\in\mathcal{L}} \alpha_i\beta_j G(\phi_k^1)}\right] < 0,$$

by the condition (2) in Theorem 2.5, the uniqueness of equilibrium is guaranteed. □

Proof of Theorem 4.1: We first show $\phi_l^*(\mathbf{0}) \geq \phi_k^*(\mathbf{0})$ by contradiction. Suppose $\phi_l^*(\mathbf{0}) < \phi_k^*(\mathbf{0})$, by Lemma A.1, it satisfies

$$\frac{\partial W_l\left(\phi_l^*(\mathbf{0}),\mathbf{0}\right)}{\partial\phi_l} \leq 0 \leq \frac{\partial W_k\left(\phi_k^*(\mathbf{0}),\mathbf{0}\right)}{\partial\phi_k}. \tag{7}$$

We denote $\hat{\phi}_k \triangleq \phi_l^*(\mathbf{0})$. Because CP m is non-dominant after signing an optimal contract, we have that

$$\pi_m(\phi_l^*(\mathbf{0}),\mathbf{0}) = \frac{\alpha_m\beta_n[G\left(\phi_l^*(\mathbf{0})\right) - G(0)] + \sum_{(m,t)\in\mathcal{L}_m} \alpha_m\beta_t G(0)}{\alpha_m\beta_n[G\left(\phi_l^*(\mathbf{0})\right) - G(0)] + \sum_{(s,t)\in\mathcal{L}} \alpha_s\beta_t G(0)} < 50\%. \tag{8}$$

Plus the conditions $m \succcurlyeq i$ and $n \succcurlyeq j$, i.e., $u_m \geq u_i, \alpha_m \geq \alpha_i$ and $v_n \leq v_j, \beta_n \geq \beta_j$, and Equation (5), we can derive that

$$\begin{aligned}\frac{\partial W_l\left(\phi_l^*(\mathbf{0}),\mathbf{0}\right)}{\partial\phi_l} &= u_m \frac{\alpha_m\beta_n G'(\phi_l^*(\mathbf{0}))[\sum_{(m,t)\in\mathcal{L}\backslash\mathcal{L}_m} \alpha_m\beta_t G(0)]}{\left\{\alpha_m\beta_n[G(\phi_l^*(\mathbf{0})) - G(0)] + \sum_{(s,t)\in\mathcal{L}} \alpha_s\beta_t G(0)\right\}^2} X - v_n \\ &\geq u_i \frac{\alpha_i\beta_j G'(\hat{\phi}_k)[\sum_{(i,t)\in\mathcal{L}\backslash\mathcal{L}_i} \alpha_i\beta_t G(0)]}{\left\{\alpha_i\beta_j[G(\hat{\phi}_k) - G(0)] + \sum_{(s,t)\in\mathcal{L}} \alpha_s\beta_t G(0)\right\}^2} X - v_j = \frac{\partial W_k\left(\hat{\phi}_k,\mathbf{0}\right)}{\partial\phi_k}.\end{aligned} \tag{9}$$

Combining Equation (7) and (9), we have

$$\frac{\partial W_k\left(\hat{\phi}_k,\mathbf{0}\right)}{\partial\phi_k} \leq \frac{\partial W_l\left(\phi_l^*(\mathbf{0}),\mathbf{0}\right)}{\partial\phi_l} \leq 0 \leq \frac{\partial W_k\left(\phi_k^*(\mathbf{0}),\mathbf{0}\right)}{\partial\phi_k}. \tag{10}$$

By Equation (6), $W_k(\phi_k,\mathbf{0})$ is strictly concave in the level ϕ_k and thus we have $\partial W_k\left(\hat{\phi}_k,\mathbf{0}\right)/\partial\phi_k > \partial W_k\left(\phi_k^*(\mathbf{0}),\mathbf{0}\right)/\partial\phi_k$ under $\hat{\phi}_k = \phi_l^*(\mathbf{0}) < \phi_k^*(\mathbf{0})$. This is contradictory with Equation (10). Therefore, it must satisfy $\phi_l^*(\mathbf{0}) \geq \phi_k^*(\mathbf{0})$.

We denote $\hat{\phi}_l = \phi_k^*(\mathbf{0})$. By Equation (8), we further have

$$\pi_m(\hat{\phi}_l, \mathbf{0}) = \frac{\alpha_m\beta_n[G(\hat{\phi}_l) - G(0)] + \sum_{(m,t)\in\mathcal{L}_m}\alpha_m\beta_t G(0)}{\alpha_m\beta_n[G(\hat{\phi}_l) - G(0)] + \sum_{(s,t)\in\mathcal{L}}\alpha_s\beta_t G(0)}\pi_m(\phi_l^*(\mathbf{0}), \mathbf{0}) < 50\%.$$

Plus the conditions $m \succcurlyeq i$ and $n \succcurlyeq j$, we can deduce that

$$\begin{aligned}
\Delta_l &= u_m[\pi_m(\phi_l^*(\mathbf{0}), \mathbf{0}) - \pi_m(0, \mathbf{0})] - v_n\phi_l^*(\mathbf{0}) \geq u_m[\pi_m(\hat{\phi}_l, \mathbf{0}) - \pi_m(0, \mathbf{0})] - v_n\hat{\phi}_l \\
&= u_m\left\{\frac{\alpha_m\beta_n[G(\hat{\phi}_l) - G(0)] + \sum_{(m,t)\in\mathcal{L}_m}\alpha_m\beta_t G(0)}{\alpha_m\beta_n[G(\hat{\phi}_l) - G(0)] + \sum_{(s,t)\in\mathcal{L}}\alpha_s\beta_t G(0)} - \frac{\sum_{(m,t)\in\mathcal{L}_m}\alpha_m\beta_t G(0)}{\sum_{(s,t)\in\mathcal{L}}\alpha_s\beta_t G(0)}\right\} - v_n\hat{\phi}_l \\
&\geq u_i\left\{\frac{\alpha_i\beta_j[G\big(\phi_k^*(\mathbf{0})\big) - G(0)] + \sum_{(i,t)\in\mathcal{L}_i}\alpha_i\beta_t G(0)}{\alpha_i\beta_j[G\big(\phi_k^*(\mathbf{0})\big) - G(0)] + \sum_{(s,t)\in\mathcal{L}}\alpha_s\beta_t G(0)} - \frac{\sum_{(i,t)\in\mathcal{L}_i}\alpha_i\beta_t G(0)}{\sum_{(s,t)\in\mathcal{L}}\alpha_s\beta_t G(0)}\right\} - v_j\phi_k^*(\mathbf{0}) \\
&= u_i[\pi_i(\phi_k^*(\mathbf{0}), \mathbf{0}) - \pi_i(0, \mathbf{0})] - v_j\phi_k^*(\mathbf{0}) = \Delta_k
\end{aligned}$$

where the first inequality is implied by the optimality of the level $\phi_l^*(\mathbf{0})$. Therefore, the profit growths satisfies $\Delta_l \geq \Delta_k$. □

Proof of Theorem 4.2 and 4.3: For any two pairs $l = (m, n)$ and $k = (i, j)$, given the contractual levels of other pairs, by Theorem 3.1, ϕ_l^* is unique for any given ϕ_k. Thus we can write it as a function of ϕ_k, denoted by $\phi_l^*(\phi_k)$. By Theorem 2.2, $\phi_l^*(\phi_k)$ maximizes the utility $W_l(\boldsymbol{\phi})$ for any ϕ_k. Thus it is the solution of a variational inequality, denoted by VI$([0, 1], -\partial W_l(\boldsymbol{\phi})/\partial\phi_l)$. By Proposition 1.3.4 of [23], $\phi_l^*(\phi_k)$ is the solution of the variational inequality if and only if it solves the Karush-Kuhn-Tucker (KKT) system of the variational inequality, i.e., there exists a multiplier $\boldsymbol{\lambda} = (\lambda_1, \lambda_2)$ such that for any level $\phi_k \in [0, 1]$, $\big(\phi_l^*(\phi_k), \boldsymbol{\lambda}\big)$ satisfies the following equations:

$$-\frac{W_l\big(\phi_l^*(\phi_k), \boldsymbol{\phi}_{-l}\big)}{\partial\phi_l} + \lambda_1 - \lambda_2 = 0, \quad 0 \leq \lambda_1 \perp [\phi_l^*(\phi_k) - 1] \leq 0, \quad \text{and} \quad 0 \leq \lambda_2 \perp [-\phi_l^*(\phi_k)] \leq 0.$$

Furthermore, by Theorem 5.4.12 of [23], the derivative $d\phi_l^*(\phi_k)/d\phi_k$ is the unique solution of an affine variational inequality, denoted by AVI $\big(C, -\partial^2 W_l(\phi_l^*, \boldsymbol{\phi}_{-l})/\partial\phi_l\partial\phi_k, -\partial^2 W_l(\phi_l^*, \boldsymbol{\phi}_{-l})/\partial\phi_l^2\big)$, where the set C is defined by

$$C = \begin{cases} (-\infty, +\infty) & \text{if} \quad \phi_l^*(\phi_k) \in (0, 1); \\ [0, +\infty) & \text{if} \quad \lambda_2 = 0, \phi_l^*(\phi_k) = 0; \\ (-\infty, 0] & \text{if} \quad \lambda_1 = 0, \phi_l^*(\phi_k) = 1; \\ \{0\} & \text{if} \quad (\lambda_1 > 0, \phi_l^*(\phi_k) = 1) \ \text{ or } \ (\lambda_2 > 0, \phi_l^*(\phi_k) = 0). \end{cases}$$

and by Equation (3) and (4), the second partial derivatives satisfy

$$\frac{\partial^2 W_l(\phi_l^*, \boldsymbol{\phi}_{-l})}{\partial\phi_l^2} = u_m\pi_l(\phi_l^*, \boldsymbol{\phi}_{-l})[1 - \pi_m(\phi_l^*, \boldsymbol{\phi}_{-l})]X\left[\frac{G''(\phi_l)}{G(\phi_l)} - 2\pi_l(\phi_l^*, \boldsymbol{\phi}_{-l})\left(\frac{G'(\phi_l)}{G(\phi_l)}\right)^2\right] < 0,$$

$$\frac{\partial^2 W_l(\phi_l^*, \boldsymbol{\phi}_{-l})}{\partial\phi_l\partial\phi_k} = \begin{cases} -2u_m[1 - \pi_m(\phi_l^*, \boldsymbol{\phi}_{-l})]\pi_l(\phi_l^*, \boldsymbol{\phi}_{-l})\dfrac{G'(\phi_l)}{G(\phi_l)}\pi_k(\phi_l^*, \boldsymbol{\phi}_{-l})\dfrac{G'(\phi_k)}{G(\phi_k)}X & \text{if} \quad k \in \mathcal{L}^m; \\ u_m[2\pi_m(\phi_l^*, \boldsymbol{\phi}_{-l}) - 1]\pi_l(\phi_l^*, \boldsymbol{\phi}_{-l})\dfrac{G'(\phi_l)}{G(\phi_l)}\pi_k(\phi_l^*, \boldsymbol{\phi}_{-l})\dfrac{G'(\phi_k)}{G(\phi_k)}X & \text{if} \quad k \notin \mathcal{L}^m \end{cases}$$

from which we have

$$\frac{\partial^2 W_l(\phi_l^*, \boldsymbol{\phi}_{-l})}{\partial \phi_l \partial \phi_k} \begin{cases} < 0 & \text{if} \quad k \in \mathcal{L}^m; \\ < 0 & \text{if} \quad k \notin \mathcal{L}^m \text{ and } \pi_m(\phi_l^*, \boldsymbol{\phi}_{-l}) < 50\%; \\ > 0 & \text{if} \quad k \notin \mathcal{L}^m \text{ and } \pi_m(\phi_l^*, \boldsymbol{\phi}_{-l}) > 50\%. \end{cases}$$

By solving AVI$(C, -\partial^2 W_l/\partial\phi_l\partial\phi_k, -\partial^2 W_l/\partial\phi_l^2)$, we can get its unique solution as follows:
1) when the set $C = (-\infty, +\infty)$,

$$\frac{d\phi_l^*(\phi_k)}{d\phi_k} = -\frac{\partial^2 W_l(\phi_l^*, \boldsymbol{\phi}_{-l})/\partial\phi_l\partial\phi_k}{\partial^2 W_l(\phi_l^*, \boldsymbol{\phi}_{-l})/\partial\phi_l^2} \begin{cases} < 0 & \text{if } k \in \mathcal{L}^m \text{ or } \Big(k \notin \mathcal{L}^m, \pi_m\big(\phi_l^*(\phi_k), \boldsymbol{\phi}_{-l}\big) < 50\%\Big); \\ > 0 & \text{if } k \notin \mathcal{L}^m, \pi_m\big(\phi_l^*(\phi_k), \boldsymbol{\phi}_{-l}\big) > 50\%. \end{cases}$$

2) when the set $C = [0, +\infty)$,

$$\frac{d\phi_l^*(\phi_k)}{d\phi_k} = \begin{cases} 0 & \text{if } k \in \mathcal{L}^m \text{ or } \Big(k \notin \mathcal{L}^m, \pi_m\big(\phi_l^*(\phi_k), \boldsymbol{\phi}_{-l}\big) < 50\%\Big); \\ -\dfrac{\partial^2 W_l(\phi_l^*, \boldsymbol{\phi}_{-l})/\partial\phi_l\partial\phi_k}{\partial^2 W_l(\phi_l^*, \boldsymbol{\phi}_{-l})/\partial\phi_l^2} > 0 & \text{if } k \notin \mathcal{L}^m, \pi_m\big(\phi_l^*(\phi_k), \boldsymbol{\phi}_{-l}\big) > 50\%. \end{cases}$$

3) when the set $C = (-\infty, 0]$,

$$\frac{d\phi_l^*(\phi_k)}{d\phi_k} = \begin{cases} -\dfrac{\partial^2 W_l(\phi_l^*, \boldsymbol{\phi}_{-l})/\partial\phi_l\partial\phi_k}{\partial^2 W_l(\phi_l^*, \boldsymbol{\phi}_{-l})/\partial\phi_l^2} < 0 & \text{if } k \in \mathcal{L}^m \text{ or } \Big(k \notin \mathcal{L}^m, \pi_m\big(\phi_l^*(\phi_k), \boldsymbol{\phi}_{-l}\big) < 50\%\Big); \\ 0 & \text{if } k \notin \mathcal{L}^m, \pi_m\big(\phi_l^*(\phi_k), \boldsymbol{\phi}_{-l}\big) > 50\%. \end{cases}$$

4) when the set $C = \{0\}$, $d\phi_l^*(\phi_k)/d\phi_k = 0$.
Therefore, we have that if $k \in \mathcal{L}^m$, $\phi_l^*(\phi_k)$ is non-increasing in ϕ_k, and if $k \notin \mathcal{L}^m$, $\phi_l^*(\phi_k)$ is non-increasing (non-decreasing) in ϕ_k if $\pi_m\big(\phi_l^*(\phi_k), \boldsymbol{\phi}_{-l}\big) < 50\%$ ($\pi_m\big(\phi_l^*(\phi_k), \boldsymbol{\phi}_{-l}\big) > 50\%$).

Furthermore, by Theorem 2.2, we can deduce that

$$\begin{aligned} \frac{d\psi_l^*(\phi_k)}{d\phi_k} &= \frac{1}{2}\left[\frac{\partial U_m(\phi_l^*, \boldsymbol{\phi}_{-l})}{\partial\phi_l^*}\frac{d\phi_l^*(\phi_k)}{d\phi_k} + \frac{\partial U_m(\phi_l^*, \boldsymbol{\phi}_{-l})}{\partial\phi_k} - \frac{\partial U_m(0, \boldsymbol{\phi}_{-l})}{\partial\phi_k} + \frac{\partial V_n(\phi_l^*, \boldsymbol{\phi}_{-l})}{\partial\phi_l^*}\frac{d\phi_l^*(\phi_k)}{d\phi_k}\right. \\ &\left. + \frac{\partial V_n(\phi_l^*, \boldsymbol{\phi}_{-l})}{\partial\phi_k} - \frac{\partial V_n(0, \boldsymbol{\phi}_{-l})}{\partial\phi_k}\right] = \frac{1}{2}\left[u_m X\left(\frac{\partial\pi_m(\phi_l^*, \boldsymbol{\phi}_{-l})}{\partial\phi_l^*} + v_n\right)\frac{d\phi_l^*(\phi_k)}{d\phi_k}\right. \\ &\left. + u_m X\left(\frac{\partial\pi_m(\phi_l^*, \boldsymbol{\phi}_{-l})}{\partial\phi_k} - \frac{\partial\pi_m(0, \boldsymbol{\phi}_{-l})}{\partial\phi_k}\right)\right] \begin{cases} \le 0 & \text{if} \quad k \in \mathcal{L}^m; \\ \le 0 & \text{if} \quad k \notin \mathcal{L}^m, \pi_m\big(\phi_l^*(\phi_k), \boldsymbol{\phi}_{-l}\big) < 50\%; \\ \ge 0 & \text{if} \quad k \notin \mathcal{L}^m, \pi_m\big(0, \boldsymbol{\phi}_{-l}\big) > 50\%. \end{cases} \end{aligned}$$

Therefore, we have that if $k \in \mathcal{L}^m$, $\psi_l^*(\phi_k)$ is non-increasing in ϕ_k, and if $k \notin \mathcal{L}^m$, $\psi_l^*(\phi_k)$ is non-increasing (non-decreasing) in ϕ_k if $\pi_m\big(\phi_l^*(\phi_k), \boldsymbol{\phi}_{-l}\big) < 50\%$ ($\pi_m(0, \boldsymbol{\phi}_{-l}) > 50\%$). □

Proof of Corollary 4.5: Since the proof of Theorem 4.4 needs the result of Corollary 4.5, we first prove Corollary 4.5. We prove the existence of the full ordering of CPs by contradiction. For any two CPs $m, m' \in \mathcal{M}$ and AP $n \in \mathcal{N}$, if $\phi_{mn}^* > \phi_{m'n}^*$, $G'(\phi_{mn}^*) < G'(\phi_{m'n}^*)$ because $G(\phi)$ is strictly concave in ϕ. By Lemma A.1 and Definition 2.3, it satisfies $\partial W_{mn}(\boldsymbol{\phi}^*)/\partial\phi_{mn} \ge 0 \ge \partial W_{m'n}(\boldsymbol{\phi}^*)/\partial\phi_{m'n}$ from which we have

$$\frac{u_m \alpha_m [1 - \pi_m(\boldsymbol{\phi}^*)]}{u_{m'}\alpha_{m'}[1 - \pi_{m'}(\boldsymbol{\phi}^*)]} \ge \frac{G'(\phi_{m'n}^*)}{G'(\phi_{mn}^*)} > 1. \tag{11}$$

Suppose $\phi_{mn'}^* < \phi_{m'n'}^*$ for some AP $n' \in \mathcal{N}$, similarly, we can deduce

$$\frac{u_m \alpha_m [1 - \pi_m(\boldsymbol{\phi}^*)]}{u_{m'}\alpha_{m'}[1 - \pi_{m'}(\boldsymbol{\phi}^*)]} \le \frac{G'(\phi_{m'n'}^*)}{G'(\phi_{mn'}^*)} < 1$$

which is contradictory with Inequality (11). Therefore, it must satisfy $\phi^*_{mn'} \geq \phi^*_{m'n'}$ for any AP $n' \in \mathcal{N}$. Similarly, we can prove that such a full ordering of APs also exists. □

Proof of Theorem 4.4: We first prove $\phi^*_{mn} \geq \phi^*_{in}$ if $m \succcurlyeq i$ by contradiction. Suppose $\phi^*_{mn} < \phi^*_{in}$, by Corollary 4.5, it satisfies $\phi^*_{mt} \leq \phi^*_{it}$ for any $t \in \mathcal{N}$. Because $G(\phi)$ is increasing in ϕ, $G(\phi^*_{mt}) \leq G(\phi^*_{it})$ for any $t \in \mathcal{N}$ and thus we have

$$\pi_m(\boldsymbol{\phi}^*) = \alpha_m \frac{\sum_{(m,t)\in\mathcal{L}_m} \beta_t G(\phi^*_{mt})}{\sum_{(s,t)\in\mathcal{L}} \alpha_s \beta_t G(\phi^*_{st})} \leq \alpha_m \frac{\sum_{(i,t)\in\mathcal{L}_i} \beta_t G(\phi^*_{it})}{\sum_{(s,t)\in\mathcal{L}} \alpha_s \beta_t G(\phi^*_{st})} = \frac{\alpha_m}{\alpha_i} \pi_i(\boldsymbol{\phi}^*). \tag{12}$$

Because $G(\phi)$ is strictly concave in ϕ, $G'(\phi^*_{mn}) > G'(\phi^*_{in})$ if $\phi^*_{mn} < \phi^*_{in}$. By Lemma A.1 and Definition 2.3, it satisfies $\partial W_{mn}(\boldsymbol{\phi}^*)/\partial \phi_{mn} \leq 0 \leq \partial W_{in}(\boldsymbol{\phi}^*)/\partial \phi_{in}$ from which we can derive

$$\frac{u_m \alpha_m [1 - \pi_m(\boldsymbol{\phi}^*)]}{u_i \alpha_i [1 - \pi_i(\boldsymbol{\phi}^*)]} \leq \frac{G'(\phi^*_{in})}{G'(\phi^*_{mn})} < 1. \tag{13}$$

Combining Inequalities (12) and (13), we can deduce that

$$\frac{\pi_m(\boldsymbol{\phi}^*)[1 - \pi_m(\boldsymbol{\phi}^*)]}{\pi_i(\boldsymbol{\phi}^*)[1 - \pi_i(\boldsymbol{\phi}^*)]} \leq \frac{\alpha_m [1 - \pi_m(\boldsymbol{\phi}^*)]}{\alpha_i [1 - \pi_i(\boldsymbol{\phi}^*)]} < \frac{u_i}{u_m} \leq 1 \tag{14}$$

where the final inequality is implied by $m \succcurlyeq i$. Furthermore, because $u_m \geq u_i$ and $\alpha_m \geq \alpha_i$ under $m \succcurlyeq i$, by Equation (13), we have $\pi_m(\boldsymbol{\phi}^*) > \pi_i(\boldsymbol{\phi}^*) > 0$. Plus $\pi_m(\boldsymbol{\phi}^*) + \pi_i(\boldsymbol{\phi}^*) \leq 1$, it satisfies $\pi_m(\boldsymbol{\phi}^*)[1 - \pi_m(\boldsymbol{\phi}^*)] \geq \pi_i(\boldsymbol{\phi}^*)[1 - \pi_i(\boldsymbol{\phi}^*)]$ which is contradictory with Inequality (14). Thus $\phi^*_{mn} \geq \phi^*_{in}$ if $m \succcurlyeq i$.

We then prove $\phi^*_{in} \geq \phi^*_{ij}$ if $n \succcurlyeq j$. Suppose $\phi^*_{in} < \phi^*_{ij}$, then $G'(\phi^*_{in}) > G'(\phi^*_{ij})$ because $G(\phi)$ is strictly concave in ϕ. By Lemma A.1 and Definition 2.3, it satisfies $\partial W_{in}(\boldsymbol{\phi}^*)/\partial \phi_{in} \leq 0 \leq \partial W_{ij}(\boldsymbol{\phi}^*)/\partial \phi_{ij}$ from which we can derive that

$$(\beta_n v_j)/(\beta_j v_n) \leq G'(\phi^*_{ij})/G'(\phi^*_{in}) < 1. \tag{15}$$

However, under $n \succcurlyeq j$, $(\beta_n v_j)/(\beta_j v_n) \geq 1$ which is contradictory with Inequality (15). Thus $\phi^*_{in} \geq \phi^*_{ij}$ if $n \succcurlyeq j$. In summary, we have $\phi^*_{mn} \geq \phi^*_{in} \geq \phi^*_{ij}$ if $m \succcurlyeq i$ and $n \succcurlyeq j$. □

Proof of Corollary 5.1 and Theorem 5.2, 5.3 and 5.4: These four results are about the sensitivities of pairs' contractual levels and CPs' market shares with respect to the weights or per-user values of CPs. To prove them, we only need to show that the levels $\boldsymbol{\phi}^*$ and the corresponding market share π_s of any CP s are differentiable in CPs' weights and per-user values, and for any two pairs $l = (m, n), k = (i, j) \in \mathcal{L}$ satisfying $m \neq i$,

$$\frac{\partial \phi^*_l}{\partial \alpha_m} \begin{cases} \geq 0 & \text{if } \pi_m(\boldsymbol{\phi}^*) < 50\% \\ \leq 0 & \text{if } \pi_m(\boldsymbol{\phi}^*) \geq 50\%, \end{cases} \quad \frac{\partial \phi^*_k}{\partial \alpha_m} \begin{cases} \leq 0 & \text{if } \pi_i(\boldsymbol{\phi}^*) < 50\% \\ \geq 0 & \text{if } \pi_i(\boldsymbol{\phi}^*) \geq 50\%, \end{cases} \quad \frac{\partial \pi_m}{\partial \alpha_m} \geq 0,$$

$$\frac{\partial \pi_i}{\partial \alpha_m} \leq 0, \quad \frac{\partial \phi^*_l}{\partial u_m} \geq 0, \quad \frac{\partial \phi^*_k}{\partial u_m} \begin{cases} \leq 0 & \text{if } \pi_i(\boldsymbol{\phi}^*) < 50\% \\ \geq 0 & \text{if } \pi_i(\boldsymbol{\phi}^*) \geq 50\%, \end{cases} \quad \frac{\partial \pi_m}{\partial u_m} \geq 0,$$

$$\frac{\partial \pi_i}{\partial u_m} \leq 0, \quad \frac{\partial \phi^*_l}{\partial \alpha_0} \begin{cases} \leq 0 & \text{if } \pi_m(\boldsymbol{\phi}^*) < 50\% \\ \geq 0 & \text{if } \pi_m(\boldsymbol{\phi}^*) \geq 50\%, \end{cases} \quad \text{and} \quad \frac{\partial \pi_m}{\partial \alpha_0} \leq 0.$$

Next, we prove these inequalities. By Lemma A.2, $\boldsymbol{\phi}^*$ is the vector of levels under contractual equilibrium if and only if it is a Nash equilibrium. By Proposition 1.4.2 of [23], $\boldsymbol{\phi}^*$ is a Nash equilibrium if and only if it is the solution of a variational inequality, denoted by VI $\big(K, F(\boldsymbol{\phi})\big)$ where $K \triangleq [0, 1]^{|\mathcal{L}|}$ and $F(\boldsymbol{\phi}) \triangleq (-\partial W_r(\boldsymbol{\phi})/\partial \phi_r)_{r\in\mathcal{L}}$. By Proposition 1.3.4 of [23], $\boldsymbol{\phi}^*$ is the solution of the variational inequality if and only if it solves the Karush-Kuhn-Tucker (KKT) system of the variational inequality, i.e., there exists multipliers $\boldsymbol{\lambda} = (\lambda_r : r \in \mathcal{L})$ and $\boldsymbol{\nu} = (\nu_r : r \in \mathcal{L})$ such that $(\boldsymbol{\phi}^*, \boldsymbol{\lambda}, \boldsymbol{\nu})$ satisfies the following equations:

$$-\frac{W_r(\boldsymbol{\phi}^*)}{\partial \phi_r} + \lambda_r - \nu_r = 0, \;\; 0 \le \lambda_r \perp (\phi_r^* - 1) \le 0, \;\; \text{and} \;\; 0 \le \nu_r \perp (-\phi_r^*) \le 0, \;\; \forall r \in \mathcal{L}.$$

By Theorem 5.4.12 and 5.4.13 of [23], $\boldsymbol{\phi}^*$ is differentiable in α_m and the derivative $\partial\boldsymbol{\phi}^*/\partial\alpha_m$ is the unique solution of an affine variational inequality, denoted by AVI $\big(\mathcal{D}, J_{\alpha_m}F(\boldsymbol{\phi}^*), J_{\boldsymbol{\phi}}F(\boldsymbol{\phi}^*)\big)$, where the set $\mathcal{D}$ is defined by

$$\mathcal{D} = \left\{ (\rho_r : r \in \mathcal{L}) : \rho_r \in \begin{cases} (-\infty, +\infty) & \text{if} \;\; \phi_r^* \in (0,1); \\ [0, +\infty) & \text{if} \;\; \nu_r = 0, \phi_r^* = 0; \\ (-\infty, 0] & \text{if} \;\; \lambda_r = 0, \phi_r^* = 1; \\ \{0\} & \text{if} \;\; (\lambda_r > 0, \phi_r^* = 1) \;\; \text{or} \;\; (\nu_r > 0, \phi_r^* = 0) \end{cases} \right\} \tag{16}$$

and $J_{\alpha_m}F(\boldsymbol{\phi}^*)$ and $J_{\boldsymbol{\phi}}F(\boldsymbol{\phi}^*)$ are the Jacobian matrices of $F(\boldsymbol{\phi}^*)$ with respect to α_m and $\boldsymbol{\phi}$, respectively, where for any two pairs $r = (s,t)$ and $k = (i,j)$, by Equation (3) and (4), the individual items of $J_{\boldsymbol{\phi}}F(\boldsymbol{\phi}^*)$ satisfy

$$-\frac{\partial^2 W_r(\boldsymbol{\phi}^*)}{\partial\phi_r\partial\phi_k} = \begin{cases} u_s[\pi_s(\boldsymbol{\phi}^*) - 1]\pi_r(\boldsymbol{\phi}^*)\left[\frac{G''(\phi_r^*)}{G(\phi_r^*)} - 2\pi_r(\boldsymbol{\phi}^*)\left(\frac{G'(\phi_r^*)}{G(\phi_r^*)}\right)^2\right]X > 0 & \text{if } s = i, t = j; \\ 2u_s[1 - \pi_s(\boldsymbol{\phi}^*)]\pi_r(\boldsymbol{\phi}^*)\frac{G'(\phi_r^*)}{G(\phi_r^*)}\pi_k(\boldsymbol{\phi}^*)\frac{G'(\phi_k^*)}{G(\phi_k^*)}X > 0 & \text{if } s = i, t \ne j; \\ u_s[1 - 2\pi_s(\boldsymbol{\phi}^*)]\pi_r(\boldsymbol{\phi}^*)\frac{G'(\phi_r^*)}{G(\phi_r^*)}\pi_k(\boldsymbol{\phi}^*)\frac{G'(\phi_k^*)}{G(\phi_k^*)}X \begin{cases} > 0 & \text{if } \pi_s < 50\% \\ \le 0 & \text{if } \pi_s \ge 50\% \end{cases} & \text{if } s \ne i. \end{cases} \tag{17}$$

The individual items of $J_{\alpha_m}F(\boldsymbol{\phi}^*)$ satisfy

$$-\frac{\partial^2 W_r(\boldsymbol{\phi}^*)}{\partial\phi_r\partial\alpha_m} = \begin{cases} u_s[2\pi_s(\boldsymbol{\phi}^*) - 1]\frac{1 - \pi_m(\boldsymbol{\phi}^*)}{\alpha_m}\pi_r(\boldsymbol{\phi}^*)\frac{G'(\phi_r^*)}{G(\phi_r^*)}X \begin{cases} < 0 & \text{if } \pi_s < 50\% \\ \ge 0 & \text{if } \pi_s \ge 50\% \end{cases} & \text{if } \; s = m; \\ -u_s[2\pi_s(\boldsymbol{\phi}^*) - 1]\frac{\pi_m(\boldsymbol{\phi}^*)}{\alpha_m}\pi_r(\boldsymbol{\phi}^*)\frac{G'(\phi_r^*)}{G(\phi_r^*)}X \begin{cases} > 0 & \text{if } \pi_s < 50\% \\ \le 0 & \text{if } \pi_s \ge 50\% \end{cases} & \text{if } \; s \ne m. \end{cases}$$

We define a set of pairs $\mathcal{P}_{\alpha_m} \triangleq \{r = (s,t) : (\lambda_r > 0, \phi_r^* = 1) \text{ or } (\nu_r > 0, \phi_r^* = 0) \text{ or } (\nu_r = 0, \phi_r^* = 0, \pi_s(\boldsymbol{\phi}^*) < 50\%, s \ne m) \text{ or } (\nu_r = 0, \phi_r^* = 0, \pi_s(\boldsymbol{\phi}^*) \ge 50\%, s = m) \text{ or } (\lambda_r = 0, \phi_r^* = 1, \pi_s(\boldsymbol{\phi}^*) \ge 50\%, s \ne m) \text{ or } (\lambda_r = 0, \phi_r^* = 1, \pi_s(\boldsymbol{\phi}^*) < 50\%, s = m)\}$. We define two vectors of levels $\boldsymbol{\phi}_{\alpha_m} \triangleq (\phi_r : r \in \mathcal{L}\backslash\mathcal{P}_{\alpha_m})$ and $\boldsymbol{\phi}^*_{\alpha_m} \triangleq (\phi_r^* : r \in \mathcal{L}\backslash\mathcal{P}_{\alpha_m})$. We define a function $H(\boldsymbol{\phi}_{\alpha_m}) \triangleq (-\partial W_r(\boldsymbol{\phi})/\partial\phi_r)_{r\in\mathcal{L}\backslash\mathcal{P}_{\alpha_m}}$ and denote the Jacobian matrices of $H(\boldsymbol{\phi}^*_{\alpha_m})$ with respect to α_m, ϕ_r, and $\boldsymbol{\phi}_{\alpha_m}$ by $J_{\alpha_m}H(\boldsymbol{\phi}^*_{\alpha_m})$, $J_{\phi_r}H(\boldsymbol{\phi}^*_{\alpha_m})$, and $J_{\boldsymbol{\phi}_{\alpha_m}}H(\boldsymbol{\phi}^*_{\alpha_m})$, respectively. We denote sets $\tilde{\mathcal{M}}_{\alpha_m} = \{s : \exists t \in \mathcal{N} \text{ such that } (s,t) \in \mathcal{L}\backslash\mathcal{P}_{\alpha_m}\}$ and $\tilde{\mathcal{N}}^s_{\alpha_m} = \{t : (s,t) \in \mathcal{L}\backslash\mathcal{P}_{\alpha_m}\}$. For any pair $r = (s,t)$, we denote $y_{st} \triangleq -\pi_r(\boldsymbol{\phi}^*)[G'(\phi_r^*)]^2/[\big(1 - \pi_s(\boldsymbol{\phi}^*)\big)G''(\phi_r^*)G(\phi_r^*)] \ge 0$ and $Y^r_{\alpha_m} \triangleq -\pi_r(\boldsymbol{\phi}^*)G'(\phi_r^*)/[(1 + \sum_{j\in\tilde{\mathcal{N}}^s_{\alpha_m}} y_{sj})\big(1 - \pi_s(\boldsymbol{\phi}^*)\big)G''(\phi_r^*)] \in [0,1)$. For any CP s, we denote $Y^s_{\alpha_m} \triangleq (\sum_{j\in\tilde{\mathcal{N}}^s_{\alpha_m}} y_{sj})/(1 + \sum_{j\in\tilde{\mathcal{N}}^s_{\alpha_m}} y_{sj}) \in [0,1)$. By solving AVI $\big(\mathcal{D}, J_{\alpha_m}F(\boldsymbol{\phi}^*), J_{\boldsymbol{\phi}}F(\boldsymbol{\phi}^*)\big)$, we can get its unique solution as follows: For any pair $r = (s,t) \in \mathcal{P}_{\alpha_m}$, $\partial\phi_r^*/\partial\alpha_m = 0$; For any pair $r = (s,t) \in$

$\mathcal{L}\backslash\mathcal{P}_{\alpha_m}$,

$$\frac{\partial\phi_r^*}{\partial\alpha_m}=\frac{\det\left(J^r_{\boldsymbol{\phi}_{\alpha_m}}H(\boldsymbol{\phi}^*_{\alpha_m})\right)}{\det\left(J_{\boldsymbol{\phi}_{\alpha_m}}H(\boldsymbol{\phi}^*_{\alpha_m})\right)}=\begin{cases}\dfrac{[1-2\pi_m(\boldsymbol{\phi}^*)]Y^r_{\alpha_m}}{\alpha_m\pi_r(\boldsymbol{\phi}^*)}\dfrac{1-\pi_m(\boldsymbol{\phi}^*)-\sum\limits_{i\in\tilde{\mathcal{M}}_{\alpha_m}\backslash\{m\}}[2\pi_i(\boldsymbol{\phi}^*)-1]Y^i_{\alpha_m}}{1-\sum\limits_{i\in\tilde{\mathcal{M}}_{\alpha_m}}[2\pi_i(\boldsymbol{\phi}^*)-1]Y^i_{\alpha_m}} & \text{if } s=m;\\ \dfrac{[2\pi_s(\boldsymbol{\phi}^*)-1]Y^r_{\alpha_m}}{\alpha_m\pi_r(\boldsymbol{\phi}^*)}\dfrac{\pi_m(\boldsymbol{\phi}^*)-[2\pi_m(\boldsymbol{\phi}^*)-1]Y^m_{\alpha_m}}{1-\sum\limits_{i\in\tilde{\mathcal{M}}_{\alpha_m}}[2\pi_i(\boldsymbol{\phi}^*)-1]Y^i_{\alpha_m}} & \text{if } s\neq m.\end{cases} \tag{18}$$

where $J^r_{\boldsymbol{\phi}_{\alpha_m}}H(\boldsymbol{\phi}^*_{\alpha_m})$ is the matrix formed by replacing the column $J_{\phi_r}H(\boldsymbol{\phi}^*_{\alpha_m})$ of the matrix $J_{\boldsymbol{\phi}_{\alpha_m}}H(\boldsymbol{\phi}^*_{\alpha_m})$ by $-J_{\alpha_m}H(\boldsymbol{\phi}^*_{\alpha_m})$. Because $\pi_i(\boldsymbol{\phi}^*)-[2\pi_i(\boldsymbol{\phi}^*)-1]Y^i_{\alpha_m}>0$ for any CP i, we can derive that $1-\pi_m(\boldsymbol{\phi}^*)-\sum_{i\in\tilde{\mathcal{M}}_{\alpha_m}\backslash\{m\}}[2\pi_i(\boldsymbol{\phi}^*)-1]Y^i_{\alpha_m}>0$ and $1-\sum_{i\in\tilde{\mathcal{M}}_{\alpha_m}}[2\pi_i(\boldsymbol{\phi}^*)-1]Y^i_{\alpha_m}>0$. Combining with Equation (18), for any two pairs $l=(m,n),k=(i,j)\in\mathcal{L}$ satisfying $m\neq i$, we have the inequalities

$$\frac{\partial\phi_l^*}{\partial\alpha_m}\begin{cases}\geq 0 & \text{if } \pi_m(\boldsymbol{\phi}^*)<50\%\\ \leq 0 & \text{if } \pi_m(\boldsymbol{\phi}^*)\geq 50\%,\end{cases}\quad \frac{\partial\phi_k^*}{\partial\alpha_m}\begin{cases}\leq 0 & \text{if } \pi_i(\boldsymbol{\phi}^*)<50\%\\ \geq 0 & \text{if } \pi_i(\boldsymbol{\phi}^*)\geq 50\%,.\end{cases}$$

Furthermore, by Equation (4), we can deduce that for any CP $i\neq m$ and $s\in\mathcal{M}$,

$$\frac{\partial\pi_i(\boldsymbol{\phi}^*,\alpha_m)}{\partial\alpha_m}=-\frac{\pi_i(\boldsymbol{\phi}^*)\pi_m(\boldsymbol{\phi}^*)}{\alpha_m},\ \frac{\partial\pi_m(\boldsymbol{\phi}^*,\alpha_m)}{\partial\alpha_m}=\frac{\pi_m(\boldsymbol{\phi}^*)[1-\pi_m(\boldsymbol{\phi}^*)]}{\alpha_m},$$
$$\frac{\partial\pi_s(\boldsymbol{\phi}^*)}{\partial\phi_r}=\begin{cases}\dfrac{\pi_s(\boldsymbol{\phi}^*)\pi_r(\boldsymbol{\phi}^*)G'(\phi_r^*)}{G(\phi_r^*)} & \text{if } r\in\mathcal{L}_s;\\ \dfrac{\pi_s(\boldsymbol{\phi}^*)\pi_r(\boldsymbol{\phi}^*)G'(\phi_r^*)}{G(\phi_r^*)} & \text{if } r\notin\mathcal{L}_s.\end{cases} \tag{19}$$

Combining with Equation (18), we can derive that for any CP $i\neq m$,

$$\frac{\partial\pi_i}{\partial\alpha_m}=\frac{\partial\pi_i(\boldsymbol{\phi}^*,\alpha_m)}{\partial\alpha_m}+\sum_{r\in\mathcal{L}}\frac{\partial\pi_i(\boldsymbol{\phi}^*)}{\partial\phi_r}\frac{\partial\phi_r^*}{\partial\alpha_m}\leq 0,\quad \frac{\partial\pi_m}{\partial\alpha_m}=\frac{\partial\pi_m(\boldsymbol{\phi}^*,\alpha_m)}{\partial\alpha_m}+\sum_{r\in\mathcal{L}}\frac{\partial\pi_m(\boldsymbol{\phi}^*)}{\partial\phi_r}\frac{\partial\phi_r^*}{\partial\alpha_m}\geq 0.$$

Similarly, by Theorem 5.4.12 and 5.4.13 of [23], $\boldsymbol{\phi}^*$ is differentiable in u_m and $\partial\boldsymbol{\phi}^*/\partial u_m$ is the unique solution of an affine variational inequality, denoted by AVI $\left(\mathcal{D},J_{u_m}F(\boldsymbol{\phi}^*),J_{\boldsymbol{\phi}}F(\boldsymbol{\phi}^*)\right)$, where the set $\mathcal{D}$ was defined in Equation (16) and the individual items of the Jacobian matrix $J_{\boldsymbol{\phi}}F(\boldsymbol{\phi}^*)$ are shown in Equation (17). $J_{u_m}F(\boldsymbol{\phi}^*)$ is the Jacobian matrix of $F(\boldsymbol{\phi}^*)$ with respect to u_m, whose individual items satisfy

$$-\frac{\partial^2 W_r(\boldsymbol{\phi}^*)}{\partial\phi_r\partial u_m}=\begin{cases}-\pi_m(\boldsymbol{\phi}^*)[1-\pi_m(\boldsymbol{\phi}^*)]X<0 & \text{if } \ s=m\\ 0 & \text{if } \ s\neq m,\end{cases}\quad \forall\ r=(s,t)\in\mathcal{L}.$$

We define a set of pairs $\mathcal{P}_{u_m}\triangleq\{r=(s,t):(\lambda_r>0,\phi_r^*=1)$ or $(\nu_r>0,\phi_r^*=0)$ or $(\nu_r=0,\phi_r^*=0,\pi_s(\boldsymbol{\phi}^*)<50\%,s\neq m)$ or $(\lambda_r=0,\phi_r^*=1,\pi_s(\boldsymbol{\phi}^*)\geq 50\%,s\neq m)$ or $(\lambda_r=0,\phi_r^*=1,s=m)\}$. We define two vectors of levels $\boldsymbol{\phi}_{u_m}\triangleq(\phi_r:r\in\mathcal{L}\backslash\mathcal{P}_{u_m})$ and $\boldsymbol{\phi}^*_{u_m}\triangleq(\phi_r^*:r\in\mathcal{L}\backslash\mathcal{P}_{u_m})$. We define a function $H(\boldsymbol{\phi}_{u_m})\triangleq(-\partial W_r(\boldsymbol{\phi})/\partial\phi_r)_{r\in\mathcal{L}\backslash\mathcal{P}_{u_m}}$ and denote the Jacobian matrices of $H(\boldsymbol{\phi}^*_{u_m})$ with respect to u_m, ϕ_r, and $\boldsymbol{\phi}_{u_m}$ by $J_{u_m}H(\boldsymbol{\phi}^*_{u_m})$, $J_{\phi_r}H(\boldsymbol{\phi}^*_{u_m})$, and $J_{\boldsymbol{\phi}_{u_m}}H(\boldsymbol{\phi}^*_{u_m})$, respectively. We denote sets $\tilde{\mathcal{M}}_{u_m}=\{s:\exists t\in\mathcal{N}$ such that $(s,t)\in\mathcal{L}\backslash\mathcal{P}_{u_m}\}$ and $\tilde{\mathcal{N}}^s_{u_m}=\{t:(s,t)\in\mathcal{L}\backslash\mathcal{P}_{u_m}\}$. For any pair $r=(s,t)$, we denote $Y^r_{u_m}\triangleq-\pi_r(\boldsymbol{\phi}^*)G'(\phi_r^*)/[(1+\sum_{j\in\tilde{\mathcal{N}}^s_{u_m}}y_{sj})\left(1-\pi_s(\boldsymbol{\phi}^*)\right)G''(\phi_r^*)]\in[0,1)$. For any CP s, we denote $Y^s_{u_m} \triangleq (\sum_{j\in\tilde{\mathcal{N}}^s_{u_m}} y_{sj})/(1+\sum_{j\in\tilde{\mathcal{N}}^s_{u_m}} y_{sj}) \in [0,1)$. By solving AVI $\left(\mathcal{D}, J_{u_m}F(\boldsymbol{\phi}^*), J_{\boldsymbol{\phi}}F(\boldsymbol{\phi}^*)\right)$, we can get its unique solution as follows: For any pair $r=(s,t)\in\mathcal{P}_{u_m}$, $\partial\phi^*_r/\partial u_m = 0$; For any pair $r=(s,t)\in\mathcal{L}\backslash\mathcal{P}_{u_m}$,

$$\frac{\partial\phi^*_r}{\partial u_m} = \frac{\det\left(J^r_{\boldsymbol{\phi}_{u_m}}H(\boldsymbol{\phi}^*_{u_m})\right)}{\det\left(J_{\boldsymbol{\phi}_{u_m}}H(\boldsymbol{\phi}^*_{u_m})\right)} = \begin{cases} \dfrac{[1-\pi_m(\boldsymbol{\phi}^*)]Y^r_{u_m}}{u_m\pi_r(\boldsymbol{\phi}^*)} \dfrac{1-\sum_{i\in\tilde{\mathcal{M}}_{u_m}\backslash\{m\}}[2\pi_i(\boldsymbol{\phi}^*)-1]Y^i_{u_m}}{1-\sum_{i\in\tilde{\mathcal{M}}_{u_m}}[2\pi_i(\boldsymbol{\phi}^*)-1]Y^i_{u_m}} & \text{if } s=m; \\ \dfrac{[1-\pi_m(\boldsymbol{\phi}^*)]Y^r_{u_m}}{u_m\pi_r(\boldsymbol{\phi}^*)} \dfrac{[2\pi_s(\boldsymbol{\phi}^*)-1]Y^m_{u_m}}{1-\sum_{i\in\tilde{\mathcal{M}}_{u_m}}[2\pi_i(\boldsymbol{\phi}^*)-1]Y^i_{u_m}} & \text{if } s\neq m. \end{cases} \tag{20}$$

where $J^r_{\boldsymbol{\phi}_{u_m}}H(\boldsymbol{\phi}^*_{u_m})$ is the matrix formed by replacing the column $J_{\phi_r}H(\boldsymbol{\phi}^*_{u_m})$ of the matrix $J_{\boldsymbol{\phi}_{u_m}}H(\boldsymbol{\phi}^*_{u_m})$ by $-J_{u_m}H(\boldsymbol{\phi}^*_{u_m})$. Because $\pi_i(\boldsymbol{\phi}^*) - [2\pi_i(\boldsymbol{\phi}^*)-1]Y^i_{u_m} > 0$ for any CP i, we can derive that $1-\sum_{i\in\tilde{\mathcal{M}}_{u_m}\backslash\{m\}}[2\pi_i(\boldsymbol{\phi}^*)-1]Y^i_{u_m} > 0$ and $1-\sum_{i\in\tilde{\mathcal{M}}_{u_m}}[2\pi_i(\boldsymbol{\phi}^*)-1]Y^i_{u_m} > 0$. Combining with Equation (20), for any two pairs $l=(m,n), k=(i,j)\in\mathcal{L}$ satisfying $m\neq i$, we have the inequalities

$$\frac{\partial\phi^*_l}{\partial u_m} \geq 0, \quad \frac{\partial\phi^*_k}{\partial u_m}\begin{cases}\leq 0 & \text{if } \pi_i(\boldsymbol{\phi}^*) < 50\% \\ \geq 0 & \text{if } \pi_i(\boldsymbol{\phi}^*) \geq 50\%.\end{cases}$$

Furthermore, by Equation (19) and (20), we can derive that

$$\frac{\partial\pi_i}{\partial u_m} = \sum_{r\in\mathcal{L}}\frac{\partial\pi_i(\boldsymbol{\phi}^*)}{\partial\phi_r}\frac{\partial\phi^*_r}{\partial u_m} \leq 0, \quad \frac{\partial\pi_m}{\partial u_m} = \sum_{r\in\mathcal{L}}\frac{\partial\pi_m(\boldsymbol{\phi}^*)}{\partial\phi_r}\frac{\partial\phi^*_r}{\partial u_m} \geq 0.$$

Similarly, by Theorem 5.4.12 and 5.4.13 of [23], $\boldsymbol{\phi}^*$ is differentiable in α_0 and $\partial\boldsymbol{\phi}^*/\partial\alpha_0$ is the unique solution of an affine variational inequality, denoted by AVI $\left(\mathcal{D}, J_{\alpha_0}F(\boldsymbol{\phi}^*), J_{\boldsymbol{\phi}}F(\boldsymbol{\phi}^*)\right)$, where the set $\mathcal{D}$ was defined in Equation (16) and the individual items of the Jacobian matrix $J_{\boldsymbol{\phi}}F(\boldsymbol{\phi}^*)$ are shown in Equation (17). $J_{\alpha_0}F(\boldsymbol{\phi}^*)$ is the Jacobian matrix of $F(\boldsymbol{\phi}^*)$ with respect to α_0, whose individual items satisfy

$$-\frac{\partial^2 W_r(\boldsymbol{\phi}^*)}{\partial\phi_r\partial\alpha_0} = -u_s[2\pi_s(\boldsymbol{\phi}^*)-1]\frac{\pi_m(\boldsymbol{\phi}^*)}{\alpha_0}\pi_r(\boldsymbol{\phi}^*)\frac{G'(\phi^*_r)}{G(\phi^*_r)}X\begin{cases}> 0 & \text{if } \pi_s(\boldsymbol{\phi}^*) < 50\% \\ \leq 0 & \text{if } \pi_s(\boldsymbol{\phi}^*) \geq 50\%,\end{cases}$$

$\forall\, r=(s,t)\in\mathcal{L}$. We define a set of pairs $\mathcal{P}_{\alpha_0} \triangleq \{r=(s,t) : (\lambda_r > 0, \phi^*_r = 1) \text{ or } (\nu_r > 0, \phi^*_r = 0) \text{ or } (\nu_r = 0, \phi^*_r = 0, \pi_s(\boldsymbol{\phi}^*) < 50\%) \text{ or } (\lambda_r = 0, \phi^*_r = 1, \pi_s(\boldsymbol{\phi}^*) \geq 50\%)\}$. We define two vectors of levels $\boldsymbol{\phi}_{\alpha_0} \triangleq (\phi_r : r\in\mathcal{L}\backslash\mathcal{P}_{\alpha_0})$ and $\boldsymbol{\phi}^*_{\alpha_0} \triangleq (\phi^*_r : r\in\mathcal{L}\backslash\mathcal{P}_{\alpha_0})$. We define a function $H(\boldsymbol{\phi}_{\alpha_0}) \triangleq (-\partial W_r(\boldsymbol{\phi})/\partial\phi_r)_{r\in\mathcal{L}\backslash\mathcal{P}_{\alpha_0}}$ and denote the Jacobian matrices of $H(\boldsymbol{\phi}^*_{\alpha_0})$ with respect to α_0, ϕ_r, and $\boldsymbol{\phi}_{\alpha_0}$ by $J_{\alpha_0}H(\boldsymbol{\phi}^*_{\alpha_0})$, $J_{\phi_r}H(\boldsymbol{\phi}^*_{\alpha_0})$, and $J_{\boldsymbol{\phi}_{\alpha_0}}H(\boldsymbol{\phi}^*_{\alpha_0})$, respectively. We denote sets $\tilde{\mathcal{M}}_{\alpha_0} = \{s : \exists t\in\mathcal{N} \text{ such that } (s,t)\in\mathcal{L}\backslash\mathcal{P}_{\alpha_0}\}$ and $\tilde{\mathcal{N}}^s_{\alpha_0} = \{t : (s,t)\in\mathcal{L}\backslash\mathcal{P}_{\alpha_0}\}$. For any pair $r=(s,t)$, we denote $y_{st} \triangleq -\pi_r(\boldsymbol{\phi}^*)[G'(\phi^*_r)]^2/[\left(1-\pi_s(\boldsymbol{\phi}^*)\right)G''(\phi^*_r)G(\phi^*_r)] \geq 0$ and $Y^r_{\alpha_0} \triangleq -\pi_r(\boldsymbol{\phi}^*)G'(\phi^*_r)/[(1+\sum_{j\in\tilde{\mathcal{N}}^s_{\alpha_0}} y_{sj})\left(1-\pi_s(\boldsymbol{\phi}^*)\right)G''(\phi^*_r)] \in [0,1)$. For any CP s, we denote $Y^s_{\alpha_0} \triangleq (\sum_{j\in\tilde{\mathcal{N}}^s_{\alpha_0}} y_{sj})/(1+\sum_{j\in\tilde{\mathcal{N}}^s_{\alpha_0}} y_{sj}) \in [0,1)$. By solving AVI $\left(\mathcal{D}, J_{\alpha_0}F(\boldsymbol{\phi}^*), J_{\boldsymbol{\phi}}F(\boldsymbol{\phi}^*)\right)$, we can get its unique solution as follows: For any pair $r=(s,t)\in\mathcal{P}_{\alpha_0}$, $\partial\phi^*_r/\partial\alpha_0 = 0$; For any pair $r=(s,t)\in\mathcal{L}\backslash\mathcal{P}_{\alpha_0}$,

$$\frac{\partial\phi^*_r}{\partial\alpha_0} = \frac{\det\left(J^r_{\boldsymbol{\phi}_{\alpha_0}}H(\boldsymbol{\phi}^*_{\alpha_0})\right)}{\det\left(J_{\boldsymbol{\phi}_{\alpha_0}}H(\boldsymbol{\phi}^*_{\alpha_0})\right)} = \frac{[2\pi_s(\boldsymbol{\phi}^*)-1]Y^r_{\alpha_0}}{\alpha_0\pi_r(\boldsymbol{\phi}^*)}\frac{\pi_0(\boldsymbol{\phi}^*)}{1-\sum_{i\in\tilde{\mathcal{M}}_{\alpha_0}}[2\pi_i(\boldsymbol{\phi}^*)-1]Y^i_{\alpha_0}} \tag{21}$$

where $J^r_{\boldsymbol{\phi}_{\alpha_0}} H(\boldsymbol{\phi}^*_{\alpha_0})$ is the matrix formed by replacing the column $J_{\phi_r} H(\boldsymbol{\phi}^*_{\alpha_0})$ of the matrix $J_{\boldsymbol{\phi}_{\alpha_0}} H(\boldsymbol{\phi}^*_{\alpha_0})$ by $-J_{\alpha_0} H(\boldsymbol{\phi}^*_{\alpha_0})$. Because $\pi_i(\boldsymbol{\phi}^*) - [2\pi_i(\boldsymbol{\phi}^*) - 1]Y^i_{\alpha_0} > 0$ for any CP i, we can derive that $1 - \sum_{i \in \tilde{\mathcal{M}}_{\alpha_0}} [2\pi_i(\boldsymbol{\phi}^*) - 1]Y^i_{\alpha_0} > 0$. Combining with Equation (21), for any pair $l = (m, n) \in \mathcal{L}$, we have the inequalities

$$\frac{\partial \phi^*_l}{\partial \alpha_0} \begin{cases} \leq 0 & \text{if } \pi_m(\boldsymbol{\phi}^*) < 50\% \\ \geq 0 & \text{if } \pi_m(\boldsymbol{\phi}^*) \geq 50\%, \end{cases}$$

Furthermore, by Equation (4), (19) and (21), we can derive that for any CP $m \in \mathcal{M}$,

$$\frac{\partial \pi_m}{\partial \alpha_0} = \frac{\partial \pi_m(\boldsymbol{\phi}^*, \alpha_0)}{\partial \alpha_0} + \sum_{r \in \mathcal{L}} \frac{\partial \pi_m(\boldsymbol{\phi}^*)}{\partial \phi_r} \frac{\partial \phi^*_r}{\partial \alpha_0} = -\frac{\pi_m(\boldsymbol{\phi}^*)\pi_0(\boldsymbol{\phi}^*)}{\alpha_0} + \sum_{r \in \mathcal{L}} \frac{\partial \pi_m(\boldsymbol{\phi}^*)}{\partial \phi_r} \frac{\partial \phi^*_r}{\partial \alpha_0} \leq 0. \qquad \square$$

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